1. Compare numerically the density corresponding to the (asymptotic) normal approximation to the variance-stabilizing transformation for the binomial(100,.4)-example to the one- and two-term Edgeworth approximation (the Edgeworth-approximations are to the untransformed density). In other words, there is a normal approximation to a function of X/n. Transform this back to an approximate density for X, and compare to the Edgeworth expansion of the density of X.

2. Let $V \sim \chi^2_n$.

(a) Find an Edgeworth expansion to the density of V.

(b) Display the approximation for $n = 12$.

(c) Compare the Edgeworth approximation to that obtained from the variance-stabilizing transformation $\sqrt{V}$.

3. Let

$$f_n(x) = \frac{F_n(x+b_n) - F_n(x-b_n)}{2b_n}$$

where F_n is the edf of $X_1, \ldots, X_n$, assumed iid with continuous density f. Thus f_n can be thought of as an empirical density estimate.

(a) Show that $\mathbf{E} f_n(x) \to f(x)$ if $b_n \to 0$.

(b) Show that $\mathbf{Var} f_n(x) \to 0$ if $b_n \to 0$ and $nb_n \to \infty$.

(c) Show that (under suitable conditions) $(2nb_n)^{1/2}(f_n(x) - \mathbf{E} f_n(x)) \xrightarrow{d} \mathrm{N}(0, f(x))$.

(d) Find conditions under which $\mathbf{E} f_n(x)$ can be replaced by $f(x)$ in (c).

(e) Find the limiting distribution of $f_n(x)^{\frac{1}{2}}$, suitably normalized ($f_n(x)^{\frac{1}{2}} - f(x)^{\frac{1}{2}}$ is called a **rootogram**).

4. Using Stirling's formula

$$\Gamma(x) = x^{x-\frac{1}{2}}\, e^{-x}\, (2\pi)^{\frac{1}{2}}\, (1 + \frac{1}{12x} + O(\frac{1}{x^2})) \quad \text{as} \quad x \to \infty$$

show that for $y > 0$

$$\frac{\Gamma(x+y)}{\Gamma(x)} = x^y (1 - \frac{y(1-y)}{2x} + O(\frac{1}{x^2})) \quad \text{as} \quad x \to \infty.$$