1. We say that a random variable X is **stochastically larger than** a random variable X', written $X \overset{s}{>} X'$, if $\mathbf{P}(X > x) \geq \mathbf{P}(X' > x)$ for all x. Similarly, a family of cdf's on $\mathbf{R}$ is **stochastically increasing** if the distributions are distinct and $\theta < \theta'$ implies that $F(x;\theta) \geq F(x;\theta')$ for all x.

(a) Show that any location family $F(x;\theta) = F(x-\theta)$ is stochastically increasing.

(b) Show that for F_0, F_1 cdf's on the real line, $F_1(x) \leq F_0(x)$ for all x iff there are nondecreasing functions g_0 and g_1, and a r.v. V, such that $g_0(x) \leq g_1(x)$ for all x, and $g_i(V)$ has cdf F_i, $i = 0, 1$.
Hint: Use the inverse cdf's.

(c) Show that a family of distributions with MLR is stochastically increasing.
Hint: One way is to show that for a nondecreasing function h, $\mathbf{E}_\theta h(X)$ is a nondecreasing function of θ.

(d) Show that a location family with density $f(x-\theta)$ is MLR provided that $-\log f$ is convex (*ie,* the density is strongly unimodal).

2. Let $X_1, \ldots, X_n$ be iid U$(0,\theta)$. Derive a UMP size α test of $H_0\colon \theta = \theta_0$ vs. $H_1\colon \theta < \theta_0$. Is there a UMP test of $\theta = \theta_0$ vs $\theta \neq \theta_0$?

3. Let X have the discrete distribution

x	-2	-1	0	1	2
$\mathbf{P}(X = x)$	$\theta_1(1-\theta_2)$	$(\frac{1}{2}-\alpha)\dfrac{1-\theta_1}{1-\alpha}$	$\alpha\dfrac{1-\theta_1}{1-\alpha}$	$(\frac{1}{2}-\alpha)\dfrac{1-\theta_1}{1-\alpha}$	$\theta_1\theta_2$

where $0 \leq \theta_1 \leq \alpha < \frac{1}{2}$, $0 \leq \theta_2 \leq 1$, and $\alpha > 0$ is a given number. We want to test $H_0\colon \theta_1 = \alpha, \theta_2 = \frac{1}{2}$ against the general alternative given above, using a likelihood ratio test based on a single observation.

(a) Show that the size α test rejects when X=-2 or 2.

(b) Determine the power function of this test.

(c) Find a better test.

4. Consider a density $f(x;\theta)$, $\theta \in \mathbf{R}$, satisfying the regularity conditions for asymptotic normality of mles.

(a) Define $K(\theta_1,\theta_2) = \int (f(x;\theta_1) - f(x;\theta_2)) \log(f(x;\theta_1)/f(x;\theta_2))\, dx$. Show that K is a distance.

(b) Show that

$$\lim_{\theta_2 \to \theta_1} \frac{K(\theta_1,\theta_2)}{(\theta_1-\theta_2)^2} = I(\theta_1)$$

where I is the Fisher information in one observation.