1. Suppose that (X_i, Y_i) are iid survival times and censoring variables, respectively, with the Y's independent of the X's, and that we observe $Z_i = \min(X_i, Y_i)$ and $\delta_i = 1(X_i \le Y_i)$.

(a) Derive an asymptotic likelihood based test of the hypothesis that the X_i are exponential, vs. the alternative that they are Weibull distributed.

(b) Apply the test to the following remission times (in weeks) of leukemia patients treated with 6-MP (from Gehan, *Biometrika*, 1965):

6* 6 6 6 7 9* 10* 10 11* 13 16 17* 19* 20* 22 23 25* 32* 32* 34* 35*

where * indicates censoring ($\delta_i = 0$).

(c) Construct an asymptotic 95% confidence interval using the likelihood ratio statistic for testing the value of the exponential parameter and the data from (b). Compare this to a confidence interval obtained using the asymptotic normality of the mle of the exponential parameter. Comment.

2. Let X_{ij}, $i = 1, . . , 4$, $j = 1, \ldots, n$ be independent exponentials, with $X_{ij} \sim \exp(\theta_i)$. For large n, test the hypothesis that $\theta_1, \ldots, \theta_4$ are in arithmetic progression (*i.e,* that $\theta_2 - \theta_1 = \theta_3 - \theta_2 = \theta_4 - \theta_3$).

3. Let $X_1, \ldots, X_n$ be iid with density $f(x;\theta) = \theta g(x) + (1-\theta)h(x)$, where $g(x) = 5\exp(-5x)$and $h(x) = \exp(-x)$, $x \ge 0$, and $\theta \in [0, 1]$ is an unknown parameter. Find the locally most powerful test of $h_{0:}\theta = 0$ against $h_1 : \theta > 0$. Show how to perform the test approximately in large samples.

4. (a) Show that the likelihood ratio test for testing independence between components in a bivariate normal distribution rejects for large absolute values of the sample correlation coefficient r.

(b) Using that $(n-2)^{\frac{1}{2}} r/(1-r^2)^{\frac{1}{2}} \sim \mathrm{t}(n-2)$, find the critical value for the test in (a).

(c) The normal approximation to the distribution of r is rather poor for medium sized n. Find a variance stabilizing transformation for r, and deduce an asymptotic form of the test in (a) using this approximation.

(d) How well does the asymptotic χ^2-distribution of λ under the null hypothesis work for not very large n compared to the exact distribution in (b), and the approximate distribution in (c)?