1. Let $X|W = w \sim \mathrm{N}(w, 1/\eta)$, and consider testing $H_0\colon w = 0$ against $H_1\colon w \neq 0$ using a gamma prior for η under H_0, and a normal-gamma prior for (w, η) under the alternative (i.e., a prior having $w \sim N(0, 1/\tau)$ and $\eta \sim \Gamma(\alpha, \beta)$, independent of w). Compute the Laplace approximation to the Bayes factor for this situation.

2. Suppose that $X_1, \ldots, X_n$ are iid Poisson(λ). You do not know the value of λ, so you assign a gamma prior. Let I be any open interval on the positive half-line containing λ, and let $\pi_n(I)$ be the posterior probability of I. Show that $\lim_{n\to\infty} \pi_n(I) = 1$ with probability one.

3. A random variable X takes on the values $1, 2, 3, 4$ with probability distribution p_0 or p_1:

x	1	2	3	4
$p_0(x)$	.1	.3	.4	.2
$p_1(x)$	.2	.2	.2	.4

(a) Based on one observation from either $p_0(x)$ or $p_1(x)$, and a prior $(\pi_0, \pi_1) = (1/3, 2/3)$, perform a Bayes test between the two hypotheses.

(b) Show that the test in (a) can be thougth of as a Bayes rule ϕ for a loss that is 0 for the correct decision and 1 for an incorrect decision.

(c) Find a rule ϕ_0 which minimizes the sum of risks $R(0, \phi) + R(1, \phi) \equiv E_0\phi + E_1(1 - \phi)$.

(d) Suppose that P_0 and P_1 are any two distributions with densities f_0 and f_1, and suppose that ϕ_0 is the rule (test) wich minimizes $R(0, \phi) + R(1, \phi)$. Show that

$$R(0, \phi_0) + R(1, \phi_0) = \int \min(f_0(x), f_1(x))dx.$$