1. xxx

1.1. xxx

1.2. xxx

1.3. xxx

1.4.

1.5. Edgeworth expansions

Theorem 1.5.1 Let f_n be the density of $(\sum X_i - n\mu/(n\mu_2)^{\frac{1}{2}})$, where the X_i are iid with $\mu_r < \infty$ for some $r \geq 3$. Let $\gamma(t)$ be the characteristic function for X_i. If $|\gamma|^\nu$ is integrable for some $\nu \geq 1$, then f_n exists for all $n \geq \nu$ and as $n \to \infty$

$$f_n(x) = \phi(x)\left(1 + \sum_{k=3}^{r} n^{1-(k/2)} P_k(x)\right) + o(n^{1-(r/2))}$$

uniformly in x. Here

$$P_3(x) = \frac{\mu_3}{6\mu_2^{3/2}}(x^3 - 3x) \text{ corrects for skewness}$$

and

$$P_4(x) = \frac{\mu_4 - 3\mu_2^2}{24\mu_2^2}(x^4 - 6x^2 + 3) + \frac{\mu_3^2}{72\mu_2^3}(x^6 - 15x^4 + 45x^2 - 15)$$

corrects for kurtosis.

Remark: The Edgeworth correction, normally $O(n^{-\frac{1}{2}})$, is $O(n^{-1})$ at $x = 0$. The corrected density can be negative in the tails.

Theorem 1.5.2 (Large Deviations) Let$F_n(x)$ be the cdf of $(\sum X_i - n\mu)/(n\mu_2)^{\frac{1}{2}}$. Then if the X_i have an mgf defined for an open interval containing zero, and if x_n grows faster than $n^{1/6}$ but is $o(n^{1/4})$, we have

$$\frac{1 - F_n(x)}{1 - \Phi(x_n)} = \exp\left(\frac{\mu_3}{6\mu_2^{3/2}} x_n^3 / n^{\frac{1}{2}}\right)(1 + O(x_n n^{-\frac{1}{2}}).$$

1.6. Saddlepoint approximation

Let X have density f_X with mgf $M(t)$ and cgf $K(t) = \log M(t)$. The *exponential tilting* of f_X is the family of densities $g_X(x;\theta)$=$\exp(\theta x - K(\theta)) f_X(x)$. The *saddlepoint approximation* to the density of $\bar{X}$ is

$$f_{\bar{X}}(u) = \left(\frac{n}{2\pi K''(\hat{\theta})}\right)^{\frac{1}{2}} \exp(n(k(\hat{\theta}) - \hat{\theta}u))\,(1 + O(n^{-1})$$

where $\hat{\theta}$ solves $K'(\hat{\theta}) = u$. It often needs to be *renormalized* so as to integrate to one.