



NRCSE

Misalignment and use of deterministic models

Work with

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The choice of spatial scale— some questions

1. Which spatial scale is correct?
2. What if there is *spatial misalignment*?
3. How do we change from one spatial scale to another?
4. What if we have different spatial datasets that come to us on different spatial scales?
5. How do we combine data sources?

We need to be careful **not to be misled** in our inferences.

Changes of support

| Observed | Inference | Method |
|----------|-----------|----------------------|
| point | point | kriging |
| point | line | contouring |
| point | area | block kriging |
| area | point | ecological inference |
| area | area | misalignment |

Some issues in model assessment

Spatiotemporal misalignment

Grid boxes vs observations

Types of error

Measurement error and bias

Model error

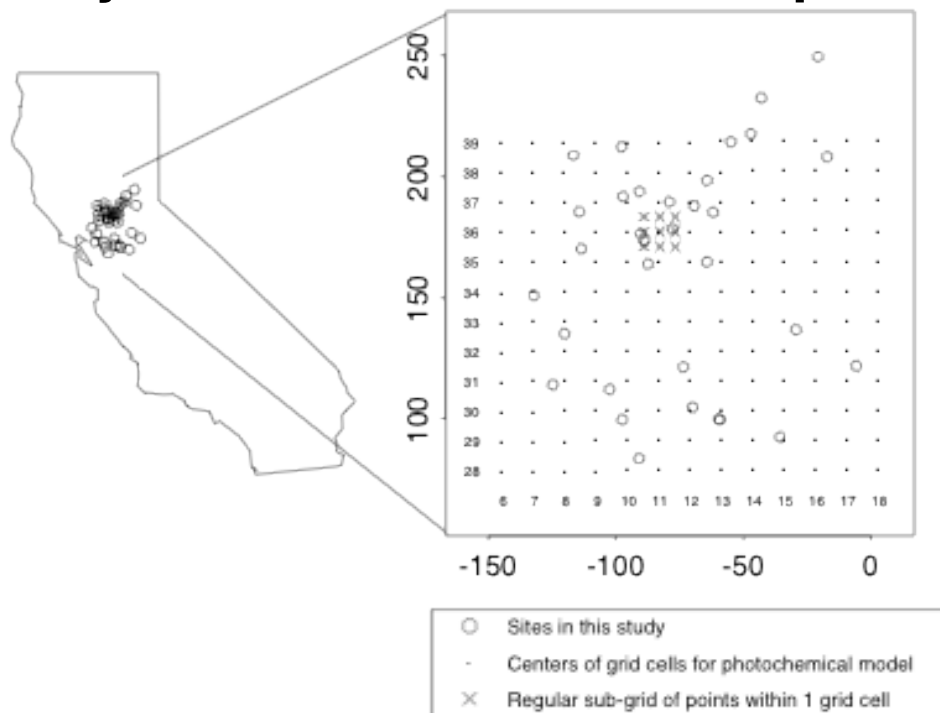
Approximation error

Manipulate data or model output?

Assessing the SARMAP model

60 days of hourly observations at 32 sites in Sacramento region

Hourly model runs for three “episodes”



Task

Estimate from data the ozone level in a grid square.

Issues:

Transformation

Diurnal cycle

Temporal dependence

Spatial dependence

Space-time interaction

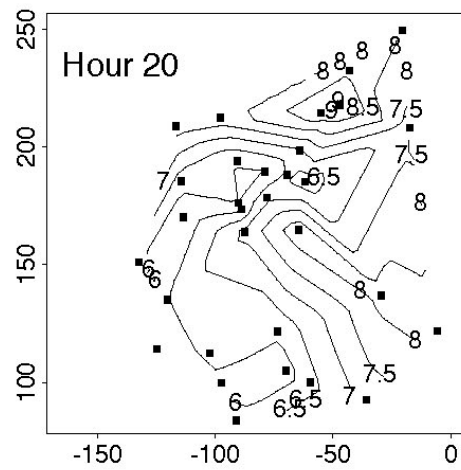
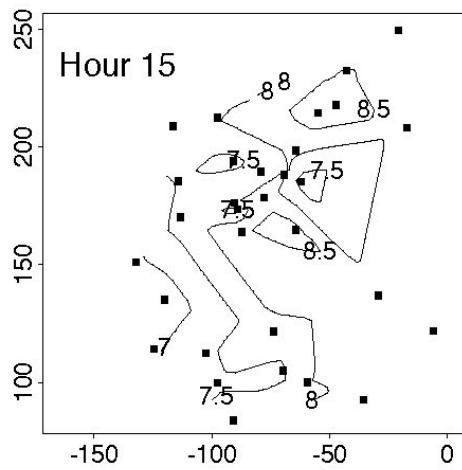
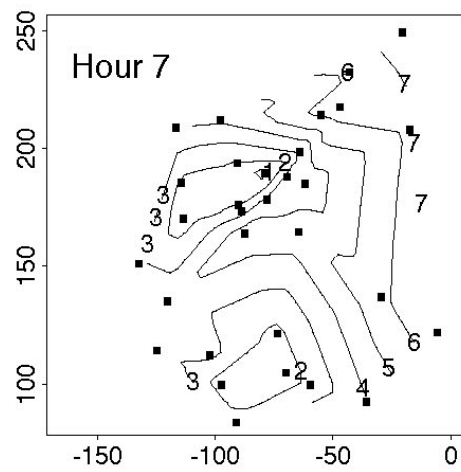
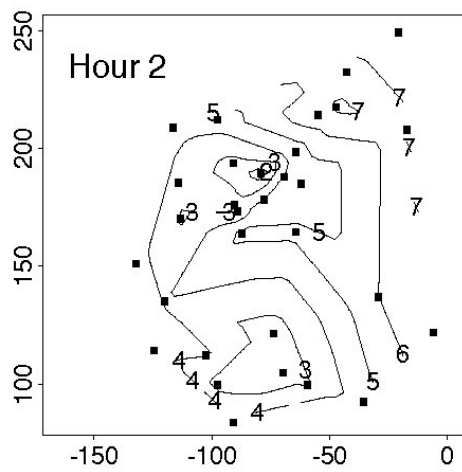
Transformation

Heterogeneous variability—mean and variance positively related

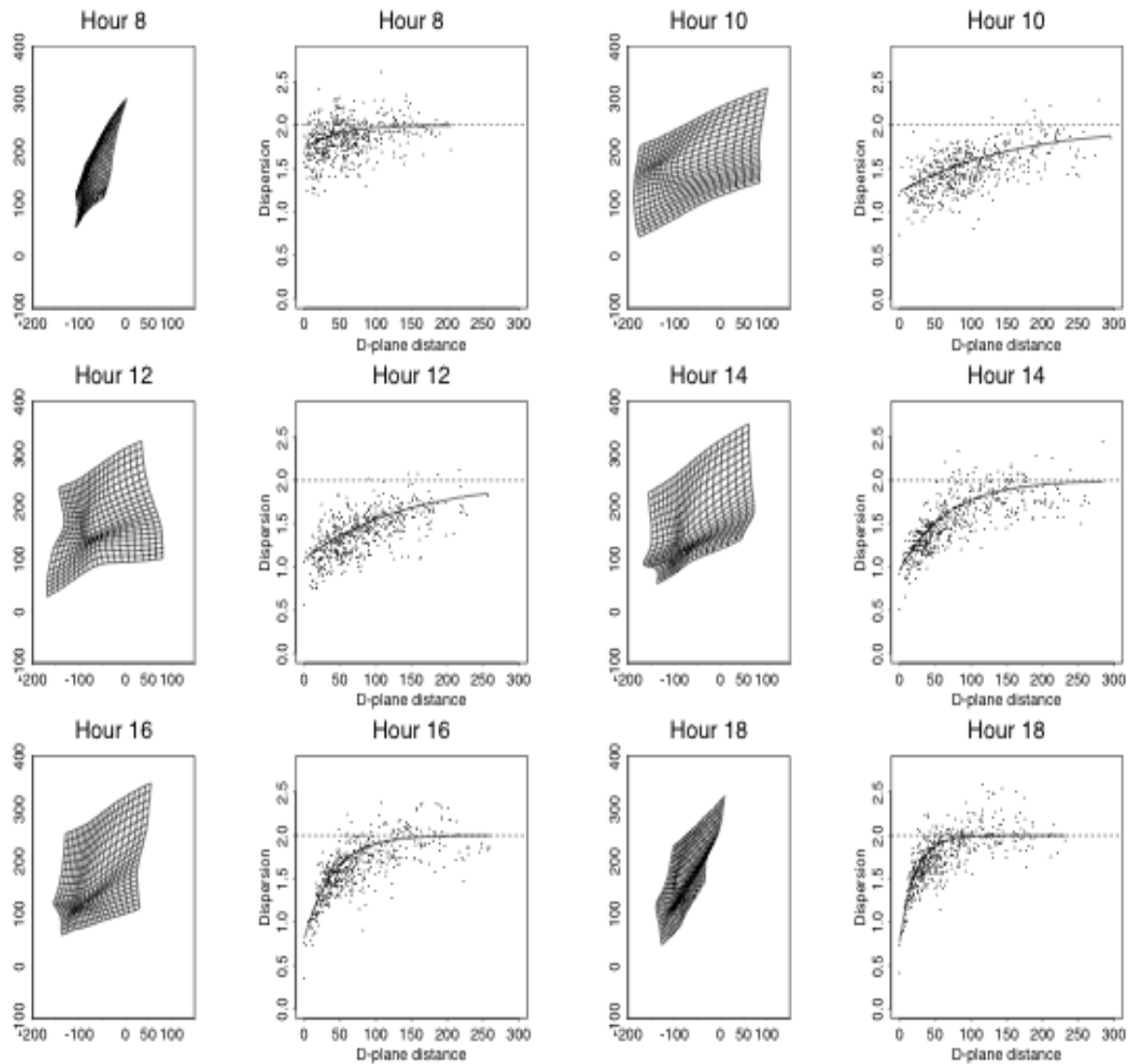
Square root transformation

All modeling now on square root scale—approximately normal

Diurnal cycle



Spatial dependence



Estimating a grid square average

$$V_t(\mathbf{s}) = \sqrt{Z_t(\mathbf{s})}$$

$$V_t(\mathbf{s}) = \mu_t(\mathbf{s}) + W_t(\mathbf{s})$$

$$W_t(\mathbf{s}) = \alpha_1(\mathbf{s})W_{t-1}(\mathbf{s}) + \alpha_2(\mathbf{s})W_{t-2}(\mathbf{s}) + Y_t(\mathbf{s})$$

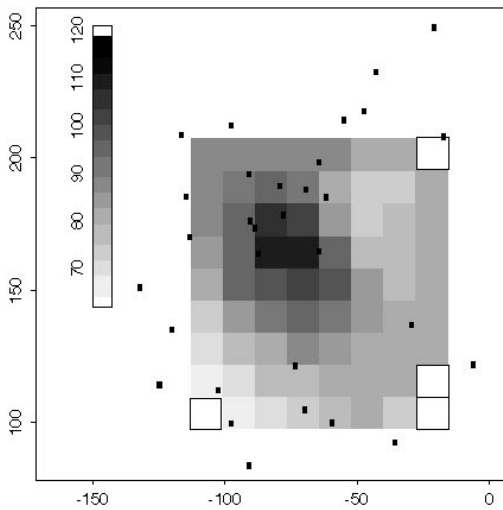
Estimate $\frac{1}{|A|} \int_A V_t^2(\mathbf{s}) ds$ using

$$\frac{1}{M} \sum E \left\{ V_t(\mathbf{s}_j)^2 \mid \text{data from } 1, \dots, t \right\}$$

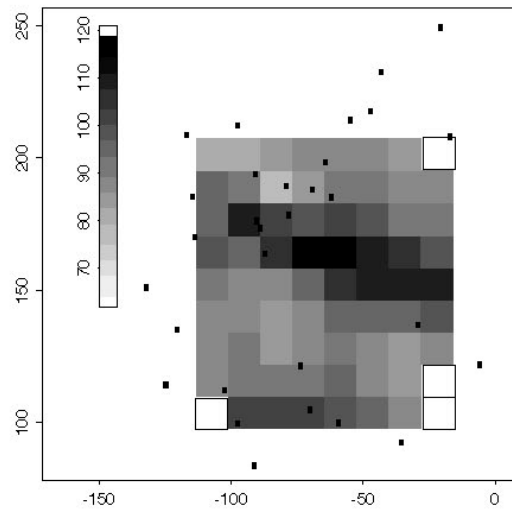
(not averages of squares of kriging estimates on the square root scale)

Afternoon comparison

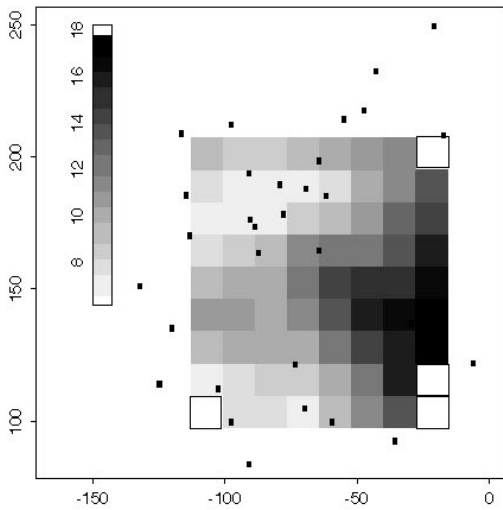
Estimated grid cell ozone levels



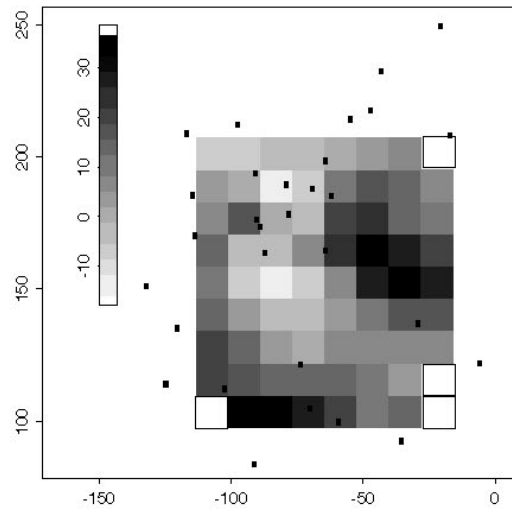
Photochemical model results



Standard error of grid cell estimates

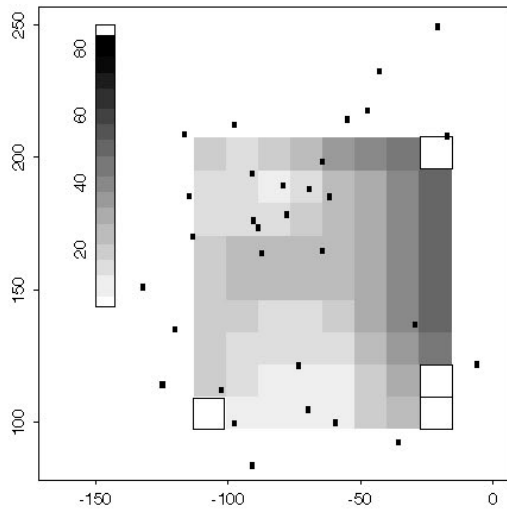


Model minus estimate

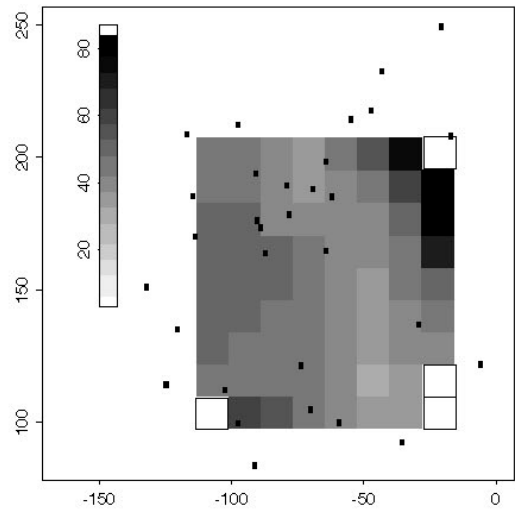


Nighttime comparison

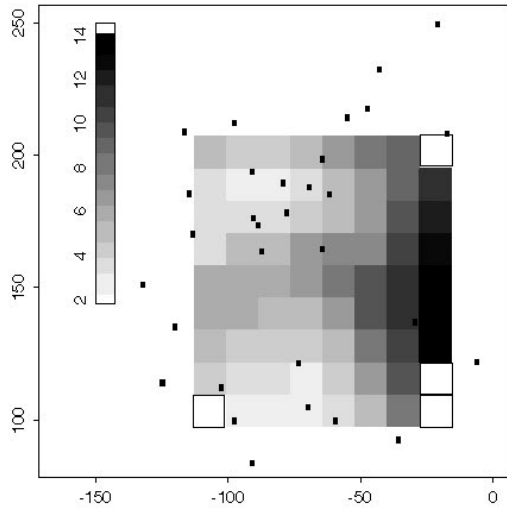
Estimated grid cell ozone levels



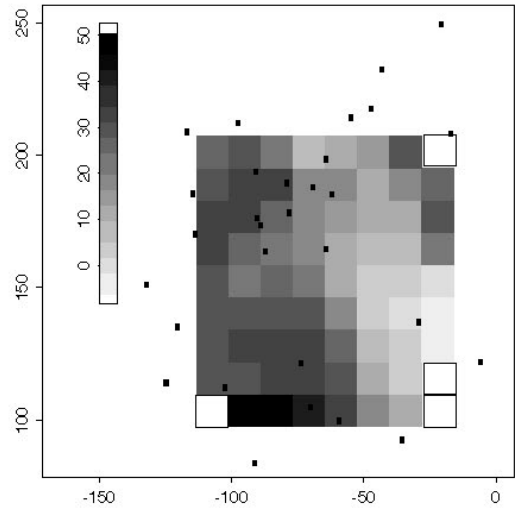
Photochemical model results



Standard error of grid cell estimates



Model minus estimate



Regional climate models

Not possible to do long runs of global models at fine resolution

Regional models (dynamic downscaling) use global model as boundary conditions and runs on finer resolution

Output is averaged over land use classes

“Weather prediction mode” uses reanalysis as boundary conditions

Comparison of model to data

**Model output daily averaged 3hr
predictions on (12.5 km)² grid**

Use open air predictions only

RCA3 driven by ERA 40/ERA Interim

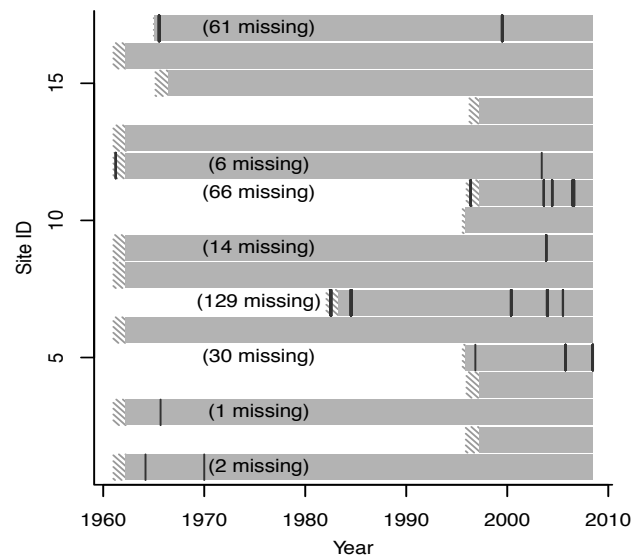
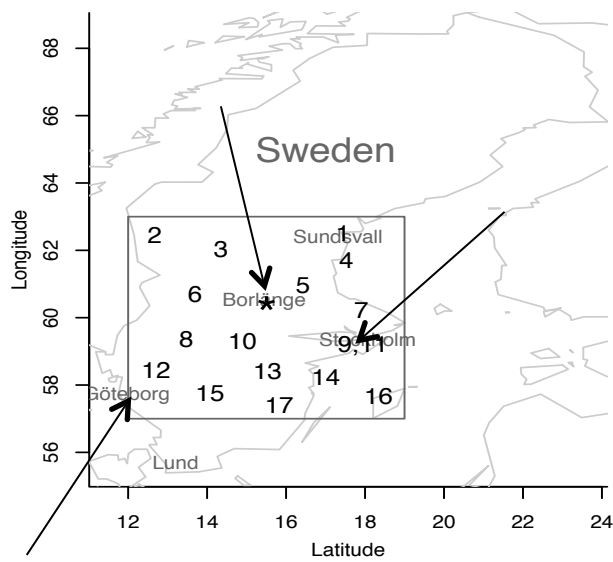
**Data daily averages point
measurements (actually weighted
average of three hourly measurements,
min and max)**

**Aggregate model and data to seasonal
averages**



Data

SMHI synoptic stations in south central Sweden, 1961-2008



Upscaling

Geostatistics: predicting grid square averages from data

Difficulties:

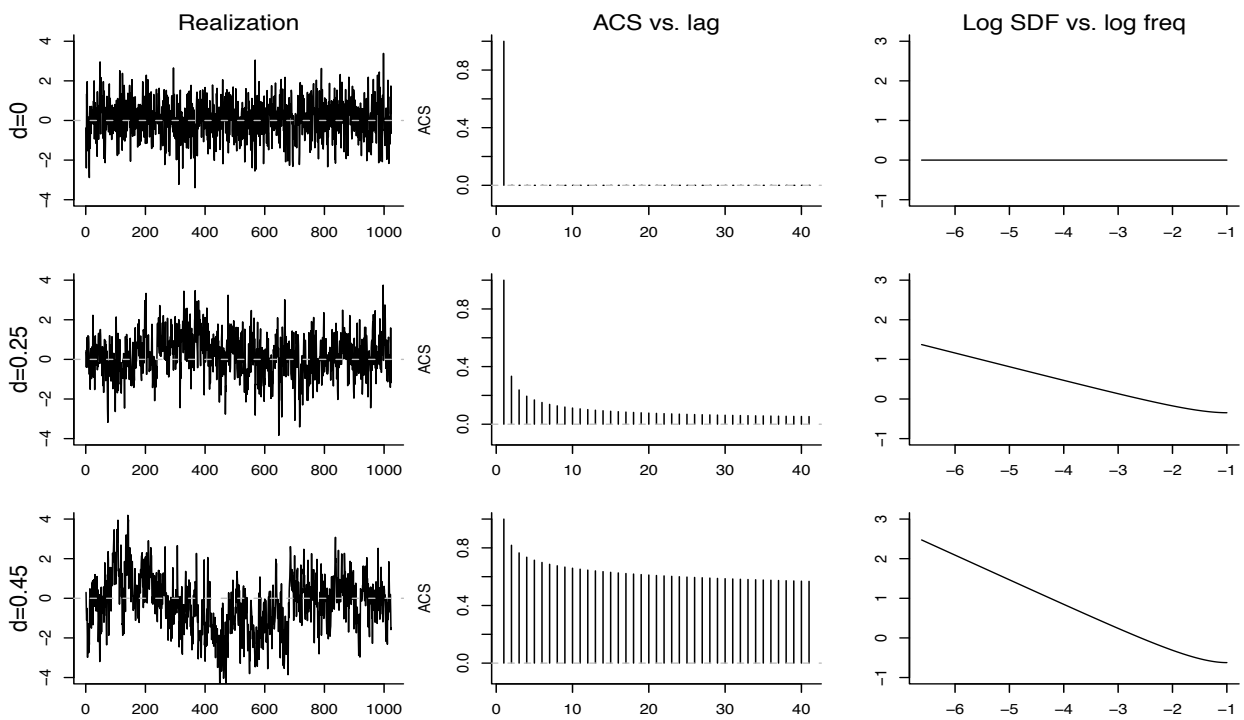
Trends

Seasonal variation

Long term memory features

Short term memory features

Long term memory models



A “simple” model

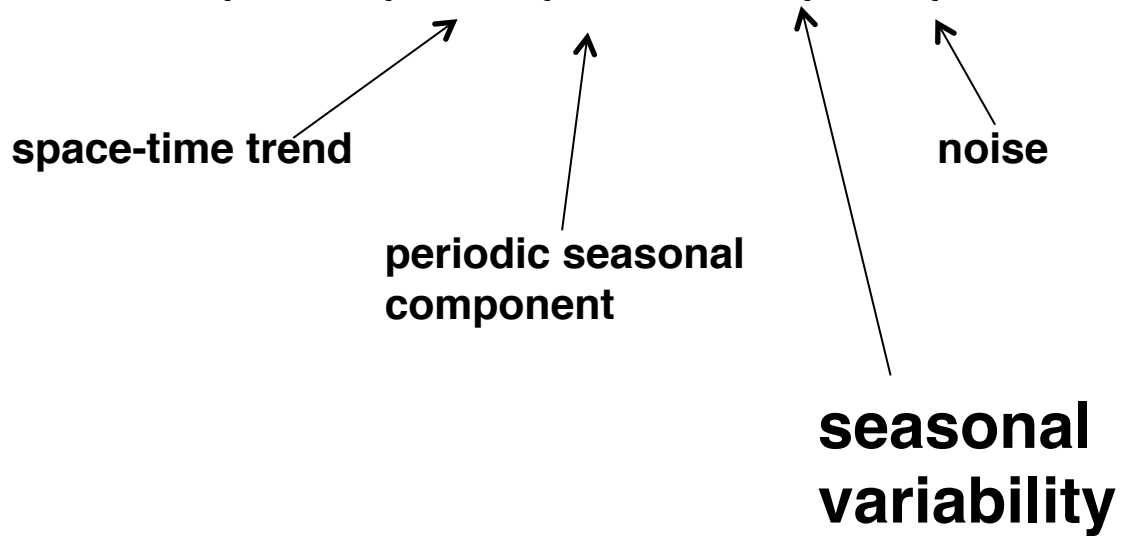
$$Y_t(\mathbf{s}) = \mu_t(\mathbf{s}) + \varphi_t(\mathbf{s}) + \exp(\alpha_t(\mathbf{s}))\eta_t(\mathbf{s})$$

space-time trend

periodic seasonal
component

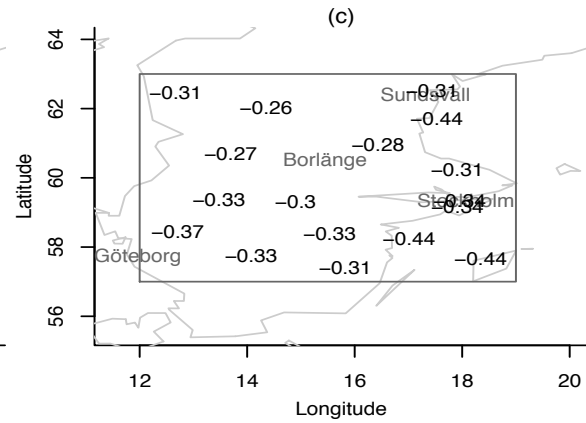
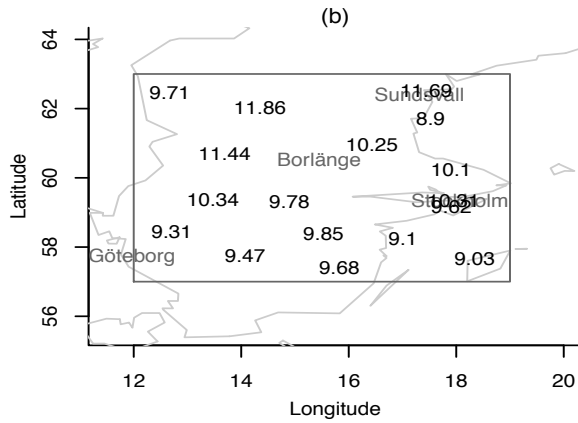
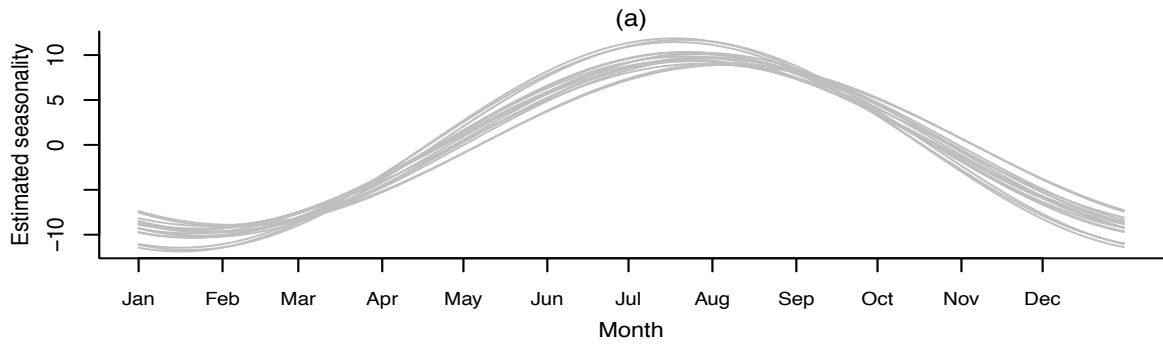
noise

seasonal
variability

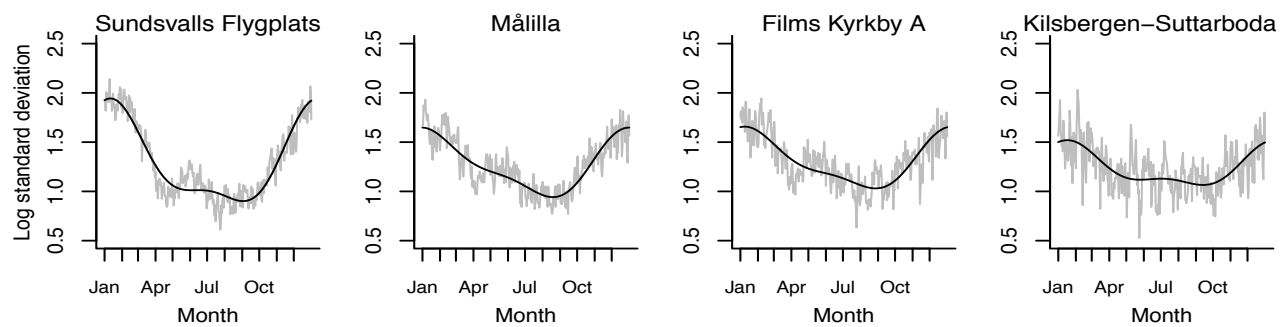


Seasonal part

$$\phi_t(\mathbf{s}) = A(\mathbf{s}) \cos(2\pi t / 365.25 + \theta(\mathbf{s}))$$



Seasonal variability



Modulate noise $\zeta_t(\mathbf{s}) = \exp(\alpha_t(\mathbf{s}))\eta_t(\mathbf{s})$
 $\alpha_t(\mathbf{s})$ two term Fourier series

Both long and short memory

Consider a stationary Gaussian process with spectral density

$$S_{\eta}(f) = B(f) |4 \sin^2(\pi f)|^{-\delta}$$

Short term
memory

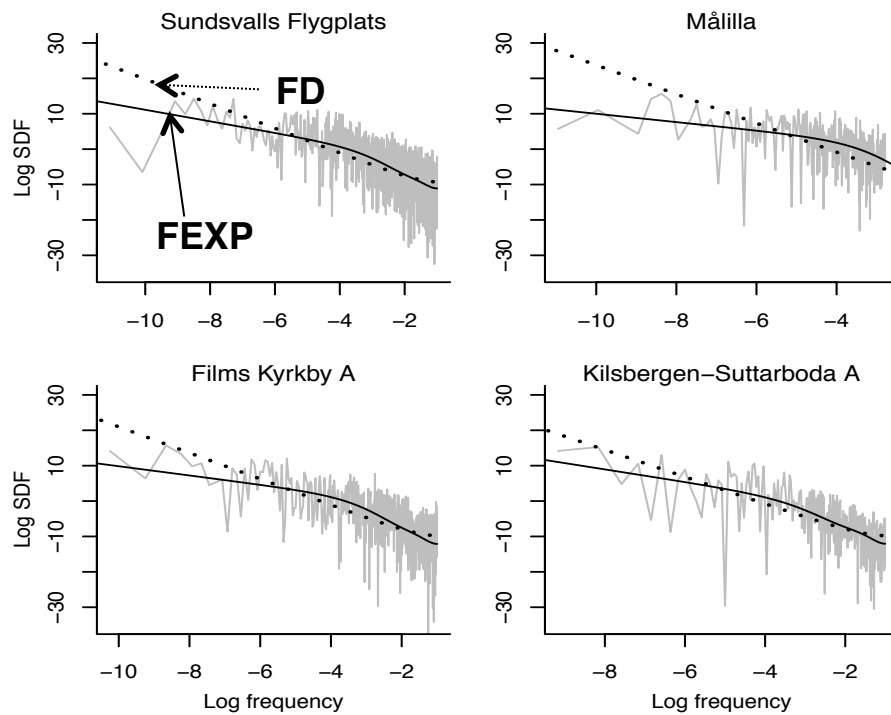
Long term
memory

Examples:

B(f) constant: fractionally differenced process (FD)

B(f) exponential: fractional exponential process (FEXP) (log B truncated Fourier series)

Estimated SDFs of standardized noise



Clear evidence of both short and long memory parts

Space-time model

Gaussian white measurement error

Process model in wavelet space

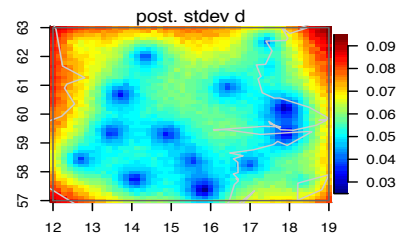
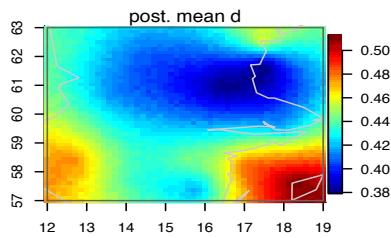
**scaling coefficients have mean linear in
time and latitude**

separable space-time covariance

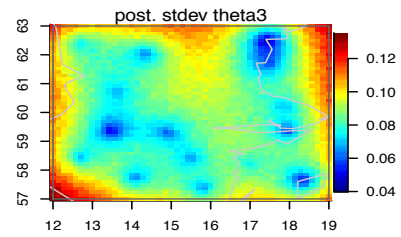
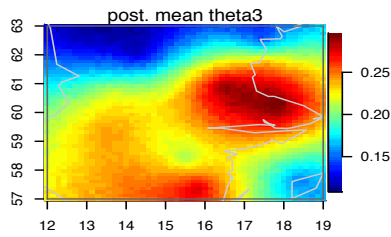
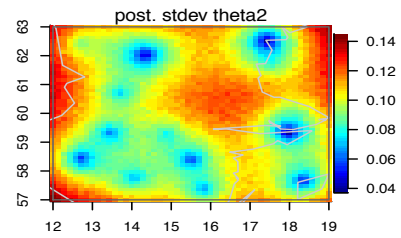
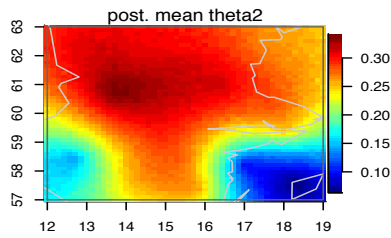
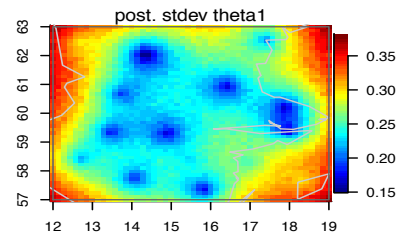
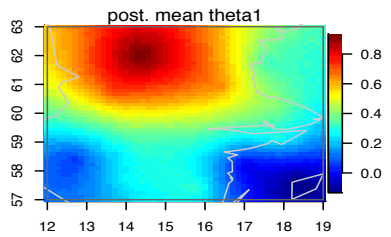
Gaussian spatially varying parameters

Dependence parameters

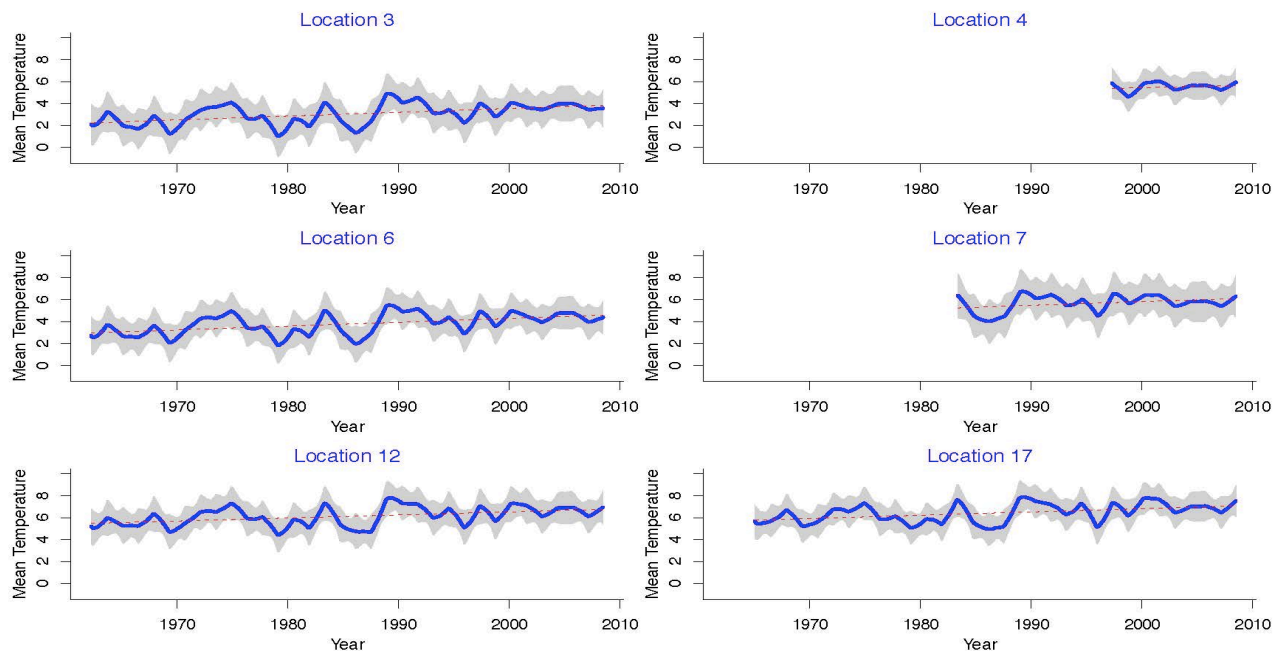
LTM



Short term



Trend estimates



Estimating grid squares

Pick q locations systematically in the grid square

Draw sample from posterior distribution of $Y(s,t)$ for s in the locations and t in the season

Compute seasonal average

Compute grid square average

Downscaling

Climatology terms:

Dynamic downscaling

Stochastic downscaling

Statistical downscaling

Here we are using the term to allow

- data assimilation for RCM**
- point prediction using RCM**

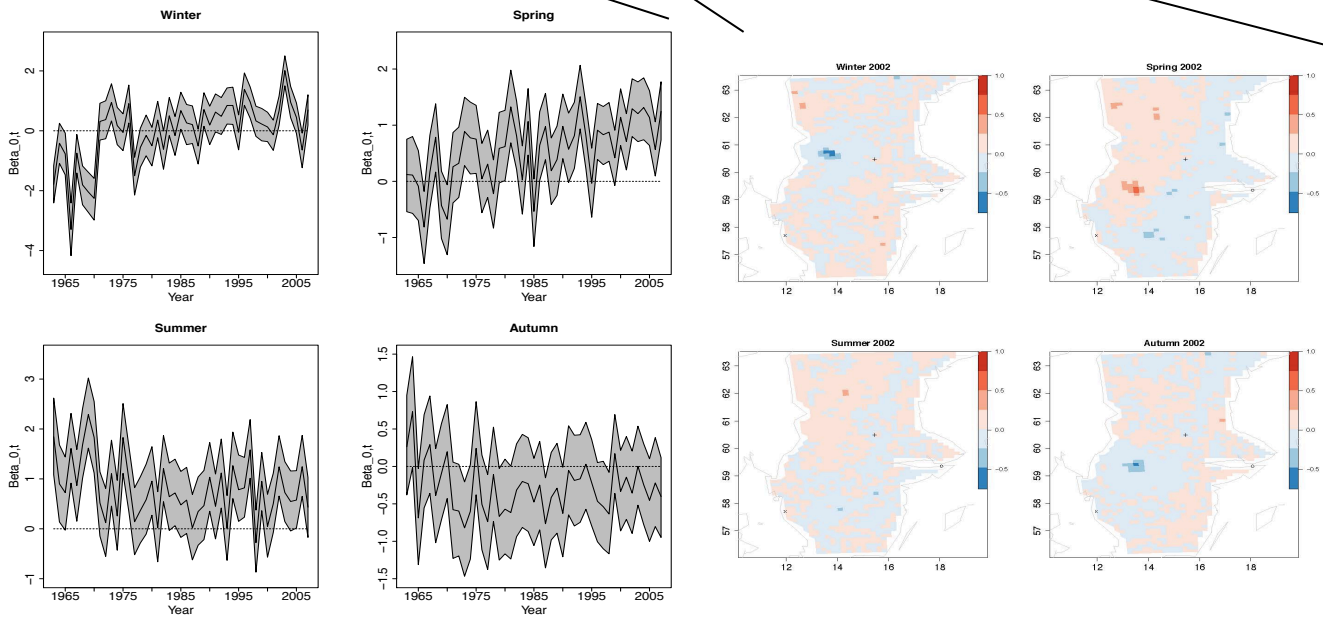
Downscaling model

$$Y(s, t) = \tilde{\beta}_0(s, t) + \beta_1 \tilde{x}(s, t) + \varepsilon(s, t)$$

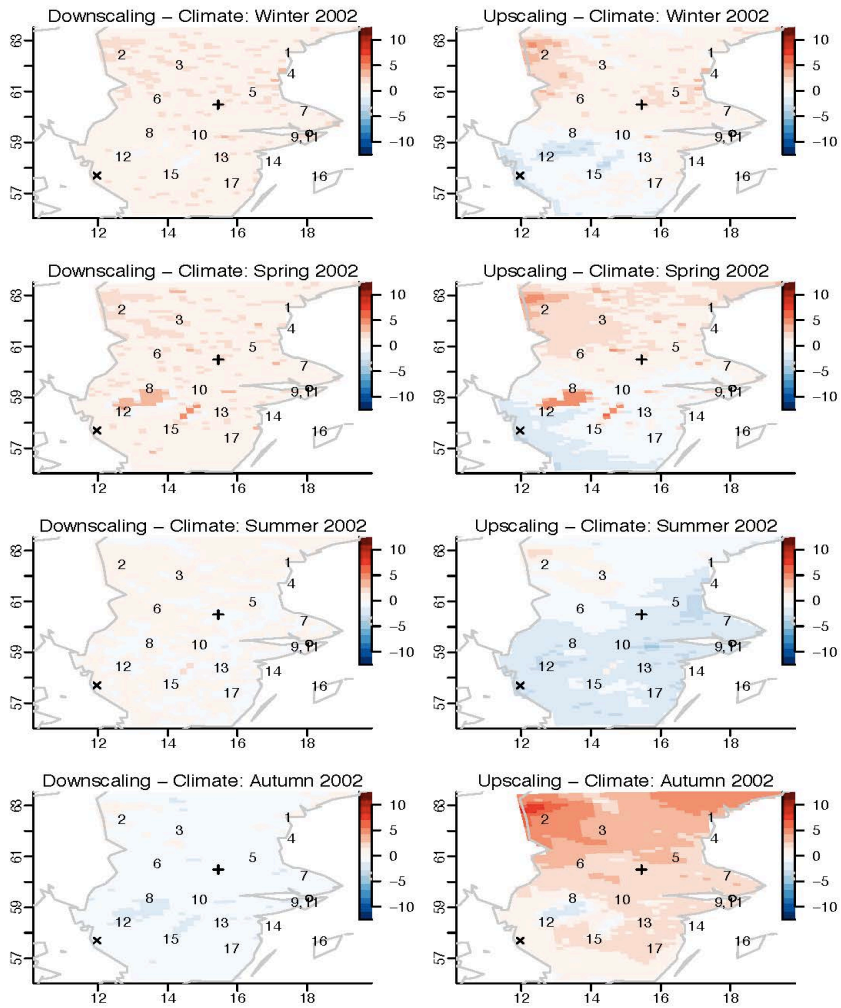
(0.91, 0.95)

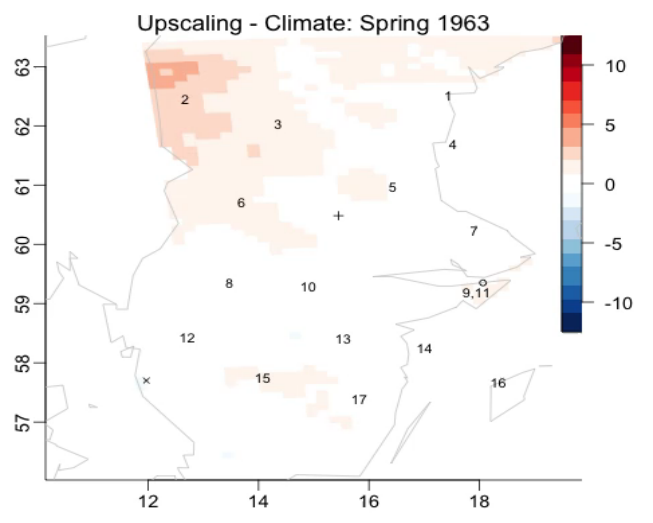
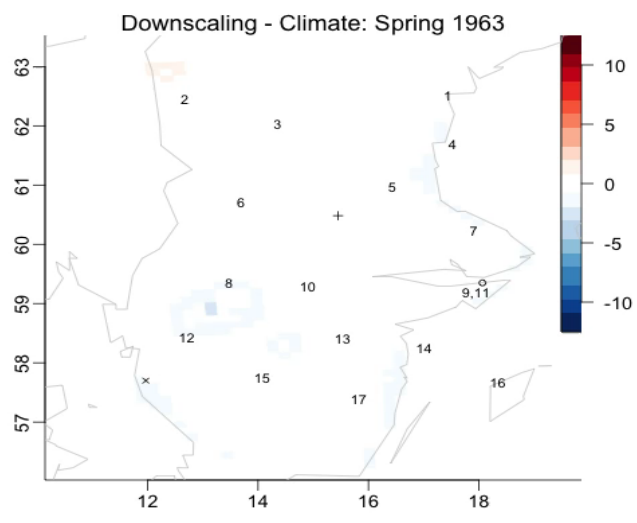
$$\tilde{\beta}_0(s, t) = \beta_0(t) + \beta(s, t)$$

smoothed
RCM



Comparisons





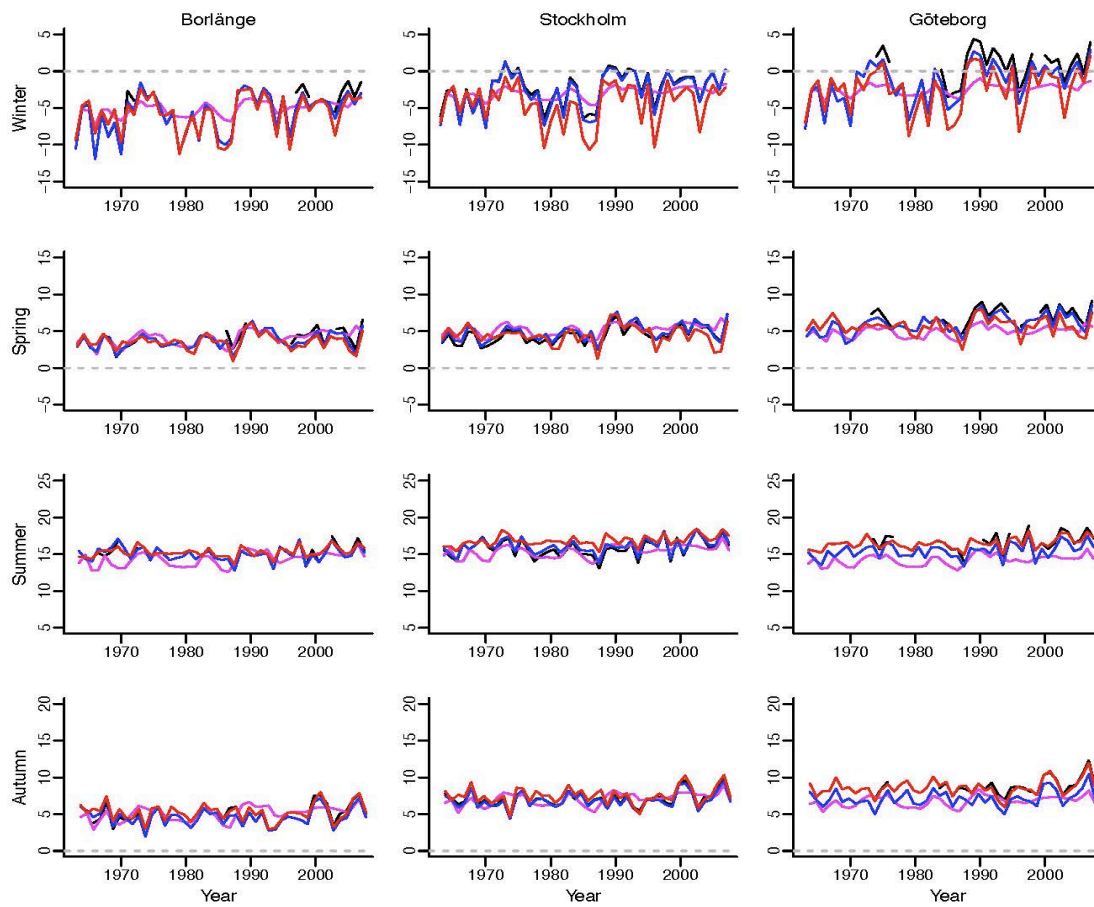
Reserved stations

Borlänge: Airport that has changed ownership, lots of missing data

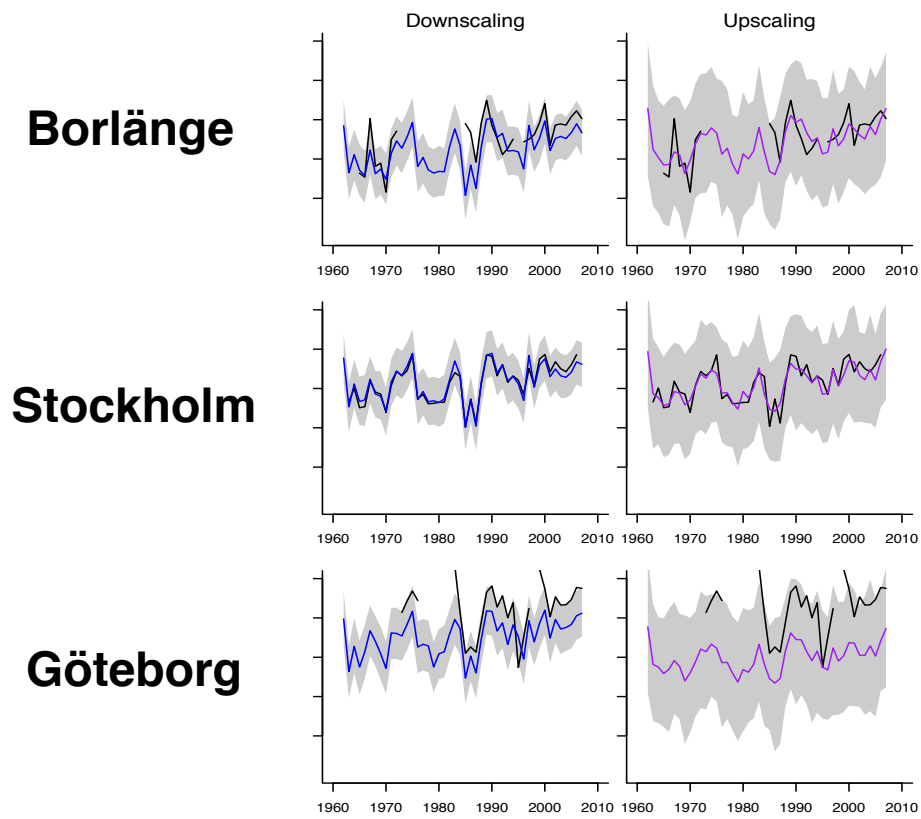
Stockholm: One of the longest temperature series in the world. Located in urban park.

Göteborg: Urban site, located just outside the grid of model output

Predictions and data



Annual scale



Comments

Nonstationarity

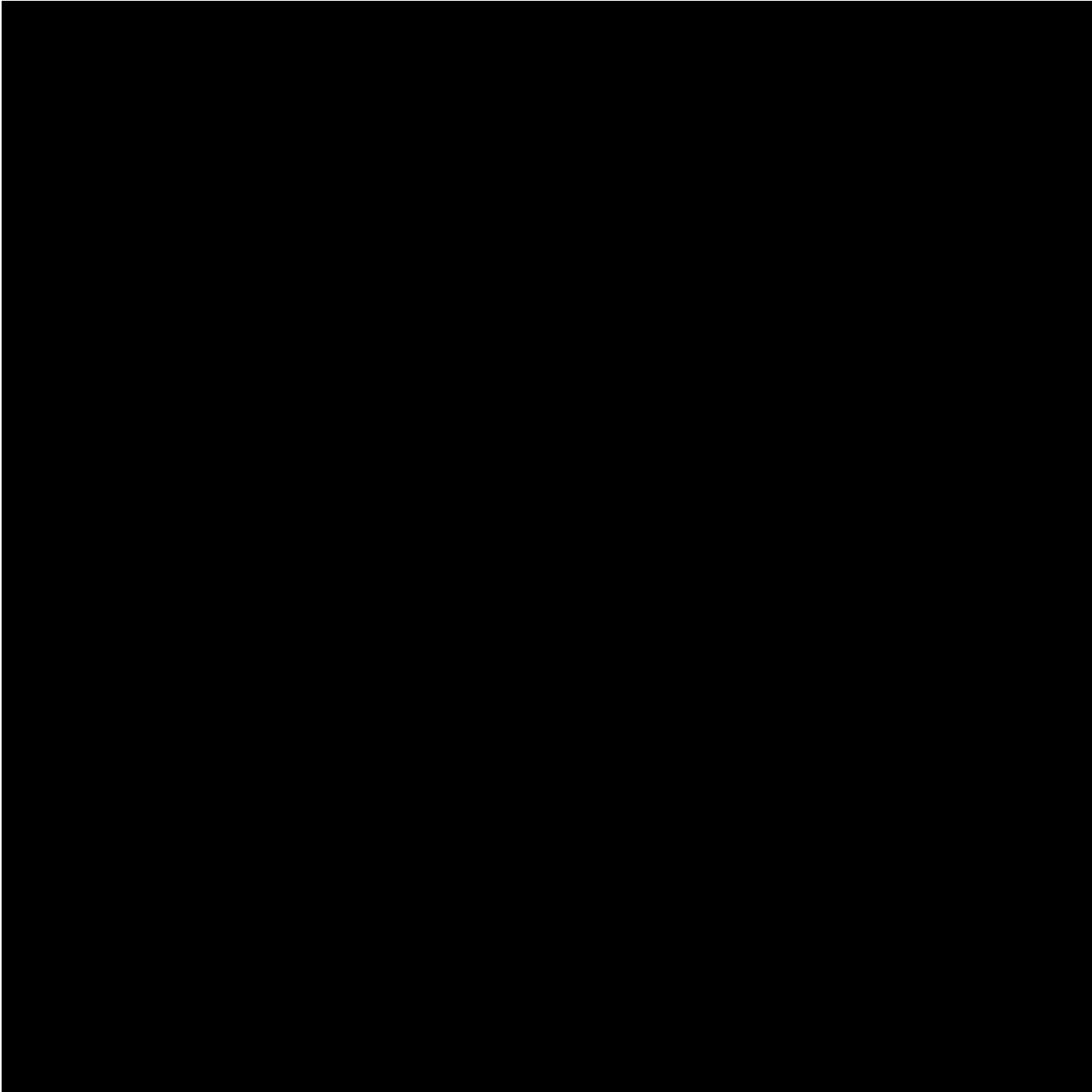
in mean

in covariance

Uncertainty in model output

**”Extreme seasons” where down-and
upscaling agree with each other but not
with the model output**

Model correction approaches



Statistical analysis of computer code output

Often the process model is expensive to run (in time, at least), especially if different runs needed for MCMC

Need to develop real-time approximation to process model

Kalman filter is a dynamic linear model approximation

SACCO is an alternative Bayesian approach

Basic framework

An emulator is a random (Gaussian) process $\eta(\mathbf{x})$ approximating the process model for input \mathbf{x} in \mathbb{R}^m .

Prior mean $m(\mathbf{x}) = \mathbf{h}(\mathbf{x})^\top \boldsymbol{\beta}$

Prior covariance $v(\mathbf{x}_1, \mathbf{x}_2) = \sigma^2 \mathbf{c}(\mathbf{x}_1, \mathbf{x}_2)$

Run the model at n input values to get n output values, so

$$(\mathbf{d} | \boldsymbol{\beta}, \sigma^2) \sim \mathbf{N}(\mathbf{H}\boldsymbol{\beta}, \sigma^2 \mathbf{C})$$

$$(\eta(\cdot) | \boldsymbol{\beta}, \sigma^2, \mathbf{d}) \sim \mathbf{N}(\mathbf{m}^*, \boldsymbol{\Sigma}^*)$$

The emulator

Integrating out β and σ^2 we get

$$\frac{\eta(\mathbf{x}) - m^{**}(\mathbf{x})}{\hat{\sigma} c^{**}(\mathbf{x}, \mathbf{x})^{1/2}} \sim t_{n-q}$$

where $q = \dim(\beta)$ and

$$m^{**}(\mathbf{x}) = \mathbf{h}(\mathbf{x})^T \hat{\beta} + \mathbf{t}(\mathbf{x})^T \mathbf{C}^{-1} (\mathbf{d} - \mathbf{H} \hat{\beta})$$

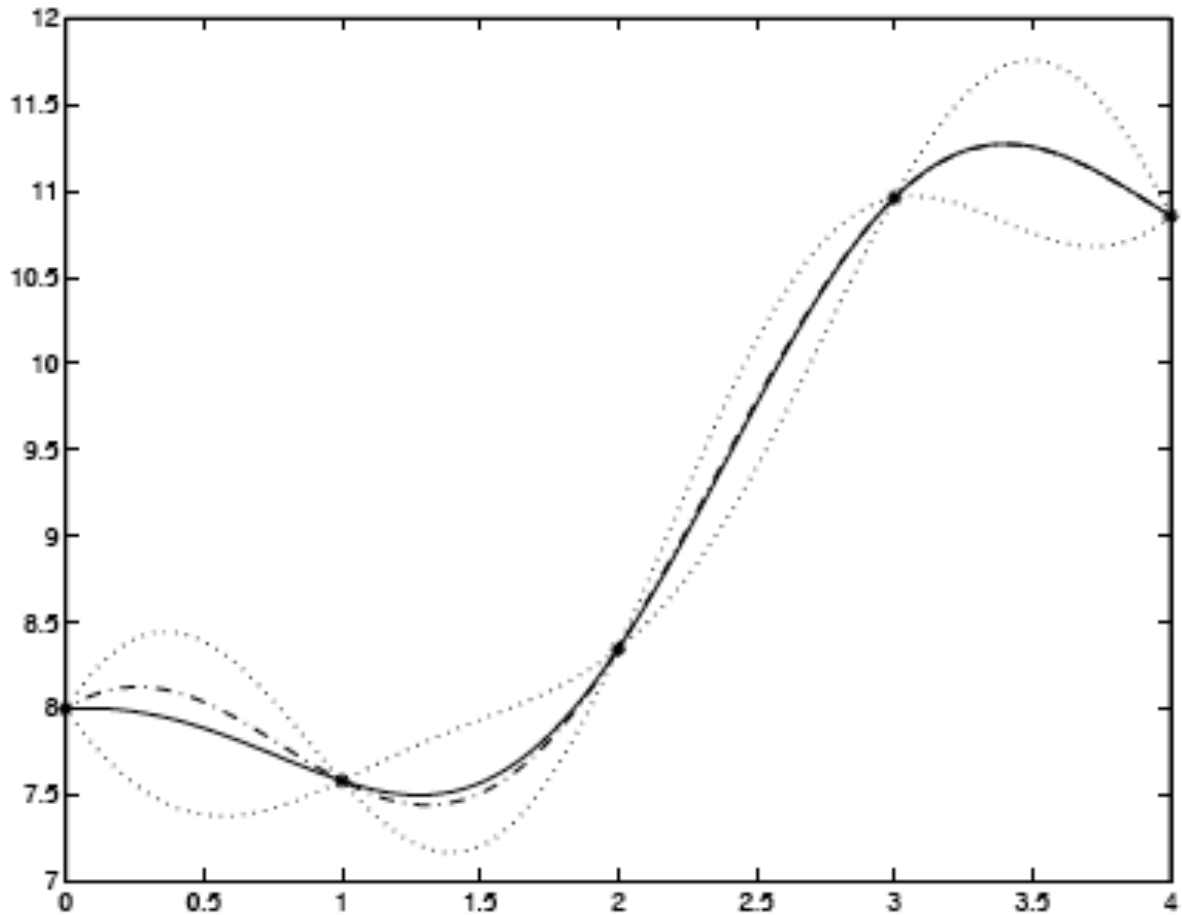
where $\mathbf{t}(\mathbf{x})^T = (\mathbf{c}(\mathbf{x}, \mathbf{x}_1), \dots, \mathbf{c}(\mathbf{x}, \mathbf{x}_n))$

m^{**} is the emulator, and we can also calculate its variance

An example

$$y=7+x+\cos(2x)$$

$$q=1, h^T(x)=(1 \ x) \ n=5$$



Conclusions

Model assessment constraints:

- **amount of data**
- **data quality**
- **ease of producing model runs**
- **degree of misalignment**

Ideally the model should have

- **similar first and second order properties to the data**
- **similar peaks and troughs to data (or simulations based on the data)**