Problem set 6.

1. Let (X,Y) be random variables with joint density $f_{X,Y}(x,y)$. Prove that

   $$E(X)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x f_{X,Y}(x,y)\,dx\,dy.$$

2. Let X be a non-negative integer-valued random variable. Show that

   $$E(X)=\sum_{k=0}^{\infty} P(X>k).$$

3. Consider an electronic system with two identical components. The system fails whenever both components fail. The manufacturer has a money-back warranty for the first 1,000 hours of operation. Let the failure times for the two components be X and Y, and assume they have joint density $\lambda^2 e^{-\lambda(x+y)}, x \geq 0, y \geq 0$ (and 0 everywhere else). As a function of λ, what is the probability that the manufacturer has to replace a system? If she wants this probability to be at most 1%, what does λ have to be?

4. Let X and Y be random variables such that for all x $F_X(x) \leq F_Y(x)$. Then X is called stochastically larger than Y. Show that $E(X) \geq E(Y)$ (assuming both expectations exist).

5. A plant releases N seeds, where $N \sim \text{Bin}(n,p)$. Each seed germinates with probability π, independently of all other seeds. Let S be the resulting number of seedlings.
   Determine
   (a) $P(S=i \mid N=k)$, the conditional mass function of S given N.
   (b) The joint pmf of S and N.
   (c) The marginal pmf of S.
   (d) The conditional pmf of N given S.