

1. Three fair dice are rolled, and the numbers are added together. The outcomes 9 and 10 can both be obtained in six different ways:

$$10 = 1 + 3 + 6 = 1 + 4 + 5 = 2 + 2 + 6 = 2 + 3 + 5 = 2 + 4 + 4 = 3 + 3 + 4$$

$$9 = 1 + 2 + 6 = 1 + 3 + 5 = 1 + 4 + 4 = 2 + 2 + 5 = 2 + 3 + 4 = 3 + 3 + 3$$

Yet, experienced gamblers claim that 10 is more likely than 9. Explain.

2. The following table contains the prediction record of a TV weather forecaster:

		Forecast		
		Sunny	Cloudy	Rainy
Actual weather	Sunny	0.25	0.05	0.05
	Cloudy	0.04	0.20	0.02
	Rainy	0.10	0.04	0.25

- What proportion of days were sunny?
- How often was the forecaster wrong?
- Given a forecast of sunny weather, what is the probability of actually having sun?
- Given that it is sunny, what is the probability that the forecaster was right?

3. Suppose $S = \{s_1, s_2, \dots\}$ is a countable sample space. Let

$$A_k = \{s_j : j \geq k\}$$

$$B = \{s_{2j+1} : j = 0, 1, 2, \dots\}$$

The probability $\mathbf{P}$ has the property that $k \mathbf{P}(A_k)$ does not depend on k .

- Show that this property determines $\mathbf{P}$ uniquely.
- Find $\mathbf{P}(B)$.

4. We throw a die twice. Consider the events

A : First throw yields 2 or 5

B : The sum of the two throws is 7

Are A and B independent?

5. A traditional final exam given by a certain professor has a 65% pass rate. A new type of test is attempted. The new test yields the same decision (pass or fail) for 80% of the students (e.g., 80% of those failing the old exam also fail the new). If a student passes the new test, what is the probability that he would have passed the old one?

6. An insurance company, which insures equal numbers of male and female drivers, estimates that a male has a probability a to make a claim in a given year (independent of other years), while a female has probability b of the same thing. The company chooses a driver at random.

- What is the probability that the chosen driver makes a claim this year?
- If the insurance company picks a claimant at random, what is the probability that the claimant will make another claim the following year?
- Find the probability that a claimant is male.