STAT 340. Autumn 2010.

1. Recall that $A \setminus B = A \cap B^c$ and $A \Delta B = (A \setminus B) \cup (B \setminus A)$. When are the following true?
    (a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
    (b) $A \cap (B \cap C) = (A \cap B) \cup C$
    (c) $A \cup (B \cup C) = A \setminus (B \setminus C)$
    (d) $(A \setminus B) \setminus C = A \setminus (B \setminus C)$
    (e) $A \Delta (B \Delta C) = (A \Delta B) \Delta C$
    (f) $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$
    (g) $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$

2. Suppose *A* and *B* are independent, and *B* and *C* are independent.
    (a) Are A and C independent (in general)?
    (b) Is *B* independent of $A \cup C$?
    (c) Is *B* independent of $A \cap C$?

3. The event *A* is called **attracted** to *B* if $P(A|B) > P(A)$. If $P(A|B) < P(A)$ then *A* is **repelled** by *B*, and if $P(A|B) = P(A)$ we say that *A* is **indifferent** to *B*.
    (a) Show that if *B* attracts *A* then *A* attracts *B* and B^c repels *A.*
    (b) A flimsy piece of paper is in one of *n* bulging file folders. The event that it is in the *j*th folder is B_j, where $P(B_j) = b_j > 0$. The event that a quick search of the *j*th folder fails to discover the paper is F_j, where $P(F_j|B_j) = f_j < 1$. Show that B_j and F_j are mutually repellent, but F_j attracts B_j for $i \neq j$. (If you look for the paper and don't find it, then it is more likely (than before you looked) to be somewhere else.)

4. Four men each tell the truth with probability 1/3. D makes a statement which C reports to B, and then B reports C's statement to A. If A claims that B denies that C claims that D is a liar, what is the probability that D spoke the truth?

5. A transmitter is sending a binary signal that must pass through three independent relay stations to get to the receiver. At each relay station, the probability that the signal is reversed (a 0 becomes a 1 or a 1 becomes a 0) is ¼. If a 0 is received, what is the probability that a 0 was sent?

6. A crooked gambler has nine dice in her coat pocket. Three are fair and six are loaded. The loaded dice each have probability ½ of showing a 6. She takes out a die at random and rolls it twice. Are the events {6 at first roll} and {6 at second roll} independent?