STAT 340 A10

Problem Set 3

1. Consider writing down outcomes for two indistinguishable dice. For example, you could treat 24 and 42 (in the usual setup where you distinguish the two dice) as the same event.
   (a) What is the sample space?
   (b) If you were to use the model that each sample point in (a) is equally likely, what would be the probability that both dice show more than a 3.
   (c) Can you suggest a different probability model for the sample space in (a) that perhaps would give more accurate predictions of long run frequencies?
   (d) How would you use a long sequence (say 25,200–a multiple both of 21 and 36) of rolls of two dice to distinguish between model (b) and (c)?
   (e) Under the model in (c), what is the probability of the event in (b)?

2. Suppose three men at a party throw their hats into a bin. The hats are mixed up, and blindfolded the men have to pick out a hat from the bin. What is the probability that none of them get their own hat? How would you solve the problem if there were n men (and hats)?

3. Ten tickets are numbered 1 through 10. Five tickets are selected one at a time with replacement. What is the probability that the highest number seen is 5?

4. Prove Boole's inequality: $P(\bigcup_{i=1}^{n} A_i) \leq \sum_{i=1}^{n} P(A_i)$.

5. Suppose you draw one card at the time from a deck of 52 cards. What is the probability that the second card drawn us an ace? How about the last card?

6. A student taking a multiple choice test either knows the answer (with probability p) or guesses between the m possible answers. Given that a student answered a question correctly, what is the probability that she knew the answer? Show that this probability is increasing both in m and p.