

Problem Set 4.

1. Consider  $n$  points on the circumference of a circle. If you pick two at random, what is the probability that they are neighbors?
2. Let  $X$  be a random variable such that  $P(X = k) = (1 - p)^{k-1}p$ . Show that  $P(X > n + k | X > n) = P(X > k)$ .

*Hint:*  $\sum_{k=0}^{\infty} a^k = \frac{1}{1-a}$  if  $|a| < 1$ .

3. The probability of infection with a certain disease in cattle is 25%. To test a newly discovered serum,  $n$  animals are injected, and the number  $k$  of infected animals is noted. Is one diseased animal out of 17 stronger or weaker evidence than none out of 10 that the serum may be working?

*Hint:* Consider a serum with no effect.

4. In how many ways can eight rooks be put on a chessboard so that none are on the main diagonal (from upper left to lower right) and no two can capture each other?
5. A certain kind of nuclear particle can split into 0, 1 or 2 *descendant* particles (it then ceases to exist), with probabilities  $\frac{1}{4}$ ,  $\frac{1}{2}$  and  $\frac{1}{4}$ , respectively. Let  $X_1$  be the number of descendants of the original particle,  $X_2$  the total number of descendants of the  $X_1$  particles etc. What is  $P(X_2 > 0)$ ?  $P(X_1 = 1 | X_2 = 1)$ ?
6. The playoff series in the National Hockey league now are all best of seven games, but the early rounds used to be best of three. If a team thinks that on any given day they have probability 0.55 to beat their first round opponent, should they prefer the three game or the seven game format?