

Problem Set 5.

1. Consider the following experiment: start by tossing a fair coin.. If it comes up heads, toss it again and record a 0 for tails or 1 for heads. But if the first toss comes up tails, choose a random number between 0 and 1 (the outcome of a random variable whose pdf is 1 between 0 and 1, 0 elsewhere; this is called the *uniform density*). Determine the cdf for the outcome of this experiment.
2. The *hazard rate* of a positive continuous random variable is $\lambda(x) = f(x) / (1 - F(x))$. It can be thought of as the conditional probability that an individual who has survived until time x fails in the next little time interval.
 - (a) Show that the exponential distribution with density $\alpha e^{-\alpha x}$ has constant hazard rate.
 - (b) By integrating both sides of the definition of hazard rate from 0 to t , show that $F(t) = 1 - \exp(-\int_0^t \lambda(x) dx)$
 - (c) A male smoker has twice the hazard rate of someone who has never smoked. Show that the probability that a smoker aged 40 survives until age 60 is the square of the probability that a non-smoker aged 40 survives until age 60.
3. Choosing a point at random in the unit circle means that all subsets of the same area have the same probability.
 - (a) What is the density for the distance from the origin to the randomly chosen point?
 - (b) What is the expected distance?
4. Let X have pmf $p_X(k)$ for any integer k . Find the pmf of
 - (a) $-X$
 - (b) $X^+ = \max(X, 0)$
 - (c) $X^- = \max(-X, 0)$
 - (d) $\text{sgn}(X) = X/|X|$ for $X \neq 0$, 0 if $X = 0$.
5. Let X be uniformly distributed on $(0,1)$, and α a given number. Find the density for $Y = -\log X / \alpha$.