STAT 341 MIDTERM EXAM Winter 2002

Name: _____ Student number: _____

Please write complete and detailed solutions. Attempt all questions. Write legibly. Additional paper is available if needed. Your are allowed one 8.5x11 sheet of paper with whatever notes you want. You are allowed to use a calculator.

1. (10 points) The random variables X and Y are independent, each taking on the values 1, 2 or 3. Complete the following table of the joint density function:

	X = 1	**X = 2**	**X = 3**
Y = 1	0.03	0.04	0.03
Y = 2	0.15		
Y = 3			

2. The triplets Peter, Paul and Patrick have identical-looking caps, except that their mother has written each triplet's name in his cap. However, the boys do not yet know how to read, sowhen they go outside they just each pick a cap at random. What is the probability that neither wears his own cap?

3. Eight people each roll two dice. Let X be the number of people who get as many ones as twos on their dice. Compute $P(X = 4)$.

4. (10 points) Let $Y = 1 / X$, where X has density $f_X(x) = (2/\pi)\ (1+x^2)^{-1}$, $x > 0$. Find the density for Y,

5. (10 points; 2 points for each correct answer, -3 for each incorrect answer. If your total score for the question is negative it will be recorded 0). For the following statements, determine whether they are true or false. Note that if a statement is not *always* true then it is false. All quantities in the question are assumed to exist. In this questions you need not show your work.

(a) $\mathrm{Var}(X + Y) + \mathrm{Var}(X - Y) = 2\,(\mathrm{Var}(X) + \mathrm{Var}(Y))$

(b) If X has a density $f_X(x)$ which is symmetric around a, so that $f_X(x - a) = f_X(a - x)$. then $\mathrm{E}(X) = a$.

(c) If $\mathrm{P}(0 \le X \le 1)$, then $\mathrm{E}(X^2) \le (\mathrm{E}(X))^2$.

(d) $\mathrm{E}(X + Y) + \mathrm{E}(X - Y) = 2\,\mathrm{E}(X)$.

(e) $\mathrm{sd}(X - Y) = \{ [\mathrm{sd}(X)]^2 + [\mathrm{sd}(Y)]^2 \}^{1/2}$ if X and Y are independent.