

STAT/MATH 491
Midterm exam

Instructions: Write complete solutions. Just stating that something is so is not sufficient, you need to say why it is so. Remember to put your name on each sheet of paper.

1. A common assumption in sociology is that the social classes of successive generations in a family follow a Markov chain. Thus, the occupation of a son is assumed to depend on the occupation of his father, but not on the occupation of his grandfather. Suppose such a model is appropriate (and does not change over time), and that the transition matrix is given by

$$P = \begin{pmatrix} 0.4 & 0.5 & 0.1 \\ 0.05 & 0.7 & 0.25 \\ 0.05 & 0.5 & 0.45 \end{pmatrix}.$$

Here the social classes are numbered 0 to 2, with 0 the highest. Father's class constitute the rows, son's class the columns. In the long run, what proportion of the population is in which social class?

2. Consider successive independent tosses of a fair coin. Suppose you are interested in the first occurrence of two successive heads.

(a) Let X_n denote the cumulative number of successive heads after the n th toss, with 2 an absorbing state. Why is it a Markov chain? What is its transition matrix P ?

(b) Express and calculate the mean number of tosses required to obtain two successive heads in terms of the chain in (a).

3. A transition matrix is called *doubly stochastic* if both rows and columns sum to 1. Show that all finite-dimensional doubly stochastic matrices have a uniform stationary distribution.

4. Consider a branching process with geometric offspring distribution (probability mass function $p(1-p)^k$, $k = 0, 1, \dots$). Determine the extinction probability.