Space-Time Modeling
Part I

this presentation borrows from presentations of Catherine Calder, Peter Guttorp, Johan Lindstrom, and Paul Sampson

Space-Time Modeling

Important resources to which we will give little attention. Any one of these could provide the basis for a full course.

Springer Series in Statistics

Statistical Analysis of Environmental Space-Time Processes

Authors: **Le**, Nhu D., **Zidek**, James V.

2006

R package: EnviroStat

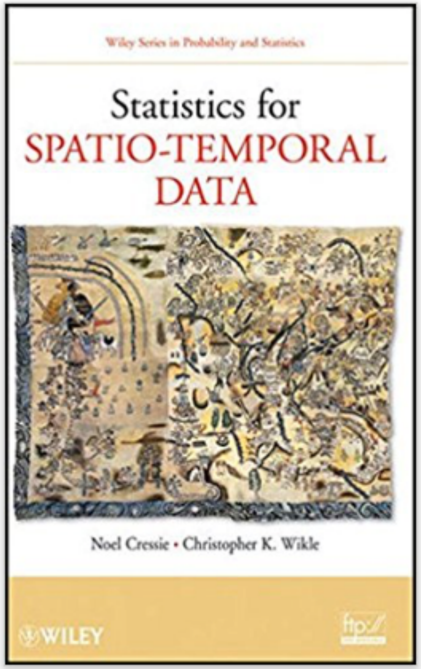
Tutorial: Chap 14

This book provides a broad introduction to the fascinating subject of environmental space-time processes; addressing the role of uncertainty. Within that context, it covers a spectrum of technical matters from measurement to environmental epidemiology to risk assessment. It showcases non-stationary vector-valued processes, while treating stationarity as a special case. The contents reflect the authors' cumulative knowledge gained over many years of consulting and research collaboration. In particular, with members of their research group, they developed within a hierarchical Bayesian framework, the new statistical approaches presented in the book for analyzing, modeling, and monitoring environmental spatio-temporal processes. Furthermore they indicate new directions for development.

This book contains technical and non-technical material and it is written for statistical scientists as well as consultants, subject area researchers and students in related fields. Novel chapters present the authors' hierarchical Bayesian approaches to

- spatially interpolating environmental processes
- designing networks to monitor environmental processes
- multivariate extreme value theory
- incorporating risk assessment.

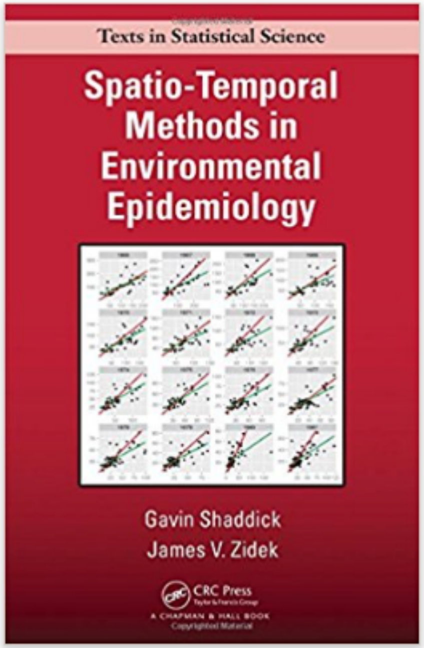
In addition, they present a comprehensive and critical survey of other approaches, highlighting deficiencies that their method seeks to overcome. Special sections marked by an asterisk provide rigorous development for readers with a strong technical background. Alternatively readers can go straight to the tutorials supplied in chapter 14 and learn how to apply the free, downloadable modeling and design software that the authors and their research partners have developed.



2011

Topics of coverage include:

- Exploratory methods for spatio-temporal data, including visualization, spectral analysis, empirical orthogonal function analysis, and LISAs
- Spatio-temporal covariance functions, spatio-temporal kriging, and time series of spatial processes
- Development of hierarchical dynamical spatio-temporal models (DSTMs), with discussion of linear and nonlinear DSTMs and computational algorithms for their implementation
- Quantifying and exploring spatio-temporal variability in scientific applications, including case studies based on real-world environmental data



Teaches Students How to Perform Spatio-Temporal Analyses within Epidemiological Studies

Spatio-Temporal Methods in Environmental Epidemiology is the first book of its kind to specifically address the interface between environmental epidemiology and spatio-temporal modeling. In response to the growing need for collaboration between statisticians and environmental epidemiologists, the book links recent developments in spatio-temporal methodology with epidemiological applications. Drawing on real-life problems, it provides the necessary tools to exploit advances in methodology when assessing the health risks associated with environmental hazards. The book's clear guidelines enable the implementation of the methodology and estimation of risks in practice.

Designed for graduate students in both epidemiology and statistics, the text covers a wide range of topics, from an introduction to epidemiological principles and the foundations of spatio-temporal modeling to new research directions. It describes traditional and Bayesian approaches and presents the theory of spatial, temporal, and spatio-temporal modeling in the context of its application to environmental epidemiology. The text includes practical examples together with embedded R code, details of specific R packages, and the use of other software, such as WinBUGS/OpenBUGS and integrated nested Laplace approximations (INLA). A supplementary website provides additional code, data, examples, exercises, lab projects, and more.

Representing a major new direction in environmental epidemiology, this book—in full color throughout—underscores the increasing need to consider dependencies in both space and time when modeling epidemiological data. Students will learn how to identify and model patterns in spatio-temporal data as well as exploit dependencies over space and time to reduce bias and inefficiency.

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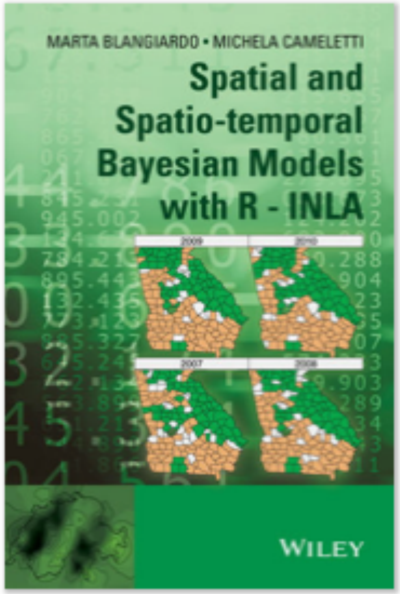
Many resources available at
https://www.stat.ubc.ca/~gavin/STEPIBookNewStyle/



2015

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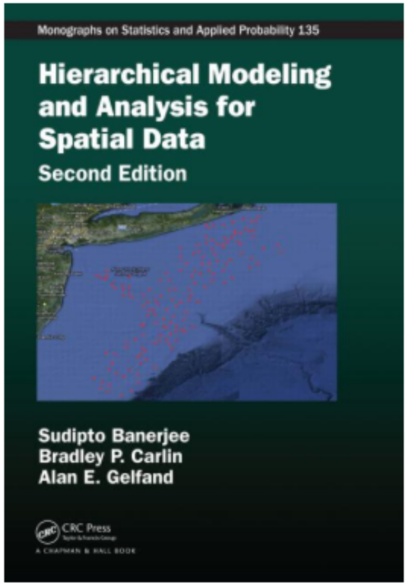

2014, 2nd edition

R package: spBayes

Hierarchical Modeling and Analysis for Spatial Data, Second Edition

Sudipto Banerjee, Bradley P. Carlin, Alan E. Gelfand

Hardback	eBook	eBook Rental
$104.95	$94.46	from $40.00

September 12, 2014 by Chapman and Hall/CRC
Reference - 584 Pages - 177 Color Illustrations
ISBN 9781439819173 - CAT# K11011
Series: Chapman & Hall/CRC Monographs on Statistics & Applied Probability

- Presents a fully model-based approach to all areas of spatial statistics, including point level, areal, and point pattern data
- Incorporates four new chapters, along with a host of updates and additions
- Offers a practical introduction to point-referenced modeling as well as some theory for those who would like more insight into the issues that arise in the geostatistical setting
- Uses up-to-date WinBUGS programs and R packages for exploratory data analysis and hierarchical modeling
- Indicates advanced/theoretical material that may be skipped by readers wanting to focus on more practical aspects
- Provides datasets and code on a supplementary website

Summary

Keep Up to Date with the Evolving Landscape of Space and Space-Time Data Analysis and Modeling

Since the publication of the first edition, the statistical landscape has substantially changed for analyzing space and space-time data. More than twice the size of its predecessor, **Hierarchical Modeling and Analysis for Spatial Data, Second Edition** reflects the major growth in spatial statistics as both a research area and an area of application.

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2017
RESeau Statistiques pour données Spatio-Temporelles
http://informatique-mia.inra.fr/resste/

Analyzing spatio-temporal data with R: Everything you always wanted to know – but were afraid to ask

Titre: Donnees spatio-temporelles avec R :
tout ce que vous avez toujours voulu savoir sans jamais avoir osé le demander

RESSTE Network et al.[1,2]

Abstract: We present an overview of (geo-)statistical models, methods and techniques for the analysis and prediction of continuous spatio-temporal processes residing in continuous space. Various approaches exist for building statistical models for such processes, estimating their parameters and performing predictions. We cover the Gaussian process approach, very common in spatial statistics and geostatistics, and we focus on R-based implementations of numerical procedures. To illustrate and compare the use of some of the most relevant packages, we treat a real-world application with high-dimensional data. The target variable is the daily mean PM_{10} concentration predicted thanks to a chemistry-transport model and observation series collected at monitoring stations across France in 2014. We give R code covering the full work-flow from importing data sets to the prediction of PM_{10} concentrations with a fitted parametric model, including the visualization of data, estimation of the parameters of the spatio-temporal covariance function and model selection. We conclude with some elements of comparison between the packages that are available today and some discussion for future developments.

Analyzing spatio-temporal data with R: Everything you always wanted to know – but were afraid to ask

Titre: Donnees spatio-temporelles avec R :
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RESSTE Network et al.[1,2]

Spatio-Temporal Modeling

SPACE-TIME DOMAINS

- Here, we assume the spatial domain is continuous.

1. continuous space and continuous time – $R^d \times R \, (= R^{d+1})$

   → Continuous (or geostatistical) space-time models

2. continuous space and discrete time – $R^d \times Z$

   → Continuous space with Discrete temporal models, or
   → Dynamic Space-Time Models

While there is considerable literature on dynamic models (see the texts by Le and Zidek, 2006, and Bannerjee et al 2014), we will not address these here.

Spatio-Temporal Modeling

CONTINUOUS SPACE-TIME MODELS

- Consider a continuous spatio-temporal random process

$$Y(\mathbf{s}, t) = \mu(\mathbf{s}, t) + \epsilon(\mathbf{s}, t), \ (\mathbf{s}, t) \in R^d \times R$$

which is indexed by $\mathbf{s} \in R^d$ in space and $t \in R$ in time.

$\rightarrow$ typically, we assume $Y(\cdot, \cdot)$ is Gaussian

---

$R^d \times R \ = R^{d+1} \dots$

- by separating a vector into its spatial and temporal components, we can use standard spatial covariance functions
- results on kriging and Gaussian process modeling in Euclidean space hold

Spatio-Temporal Modeling

Obviously, all modeling and computation is dependent on how one specifies $\mu(\mathbf{s},t)$ and $\epsilon(\mathbf{s},t)$. In general terms, there is a question of how much structure one attempts to put into the mean field or trend, $\mu(\mathbf{s},t)$, which determines the stochastic structure of $\epsilon(\mathbf{s},t)$, characterized by a space-time covariance model. Perhaps it goes without saying, but fundamental too are the spatial and temporal scales of scientific interest for modeling.

The nature of the spatio-temporal design can also influence modeling and analysis strategies.

Spatio-Temporal Modeling

- For data sparse in space but dense in time, as is typically the case with, for example, air pollution and meteorological monitoring networks, one may adopt the framework of multivariate time series analysis, although that seems not to be too common.
- For data dense in space and sparse in time, thus providing snapshots of the spatio-temporal field, one may work in a multivariate geostatistical setting where replications in time are considered as separate variables. This too does not seem to common.

Spatio-Temporal Modeling

  - The above strategies fail when data are dense in space and time, but perhaps not regular due to missing data and/or a combination of monitoring networks. Recently we have had to address data from "mobile monitoring" with sensors on vehicles or even mobile phones. These problems are challenging and require carefull consideration of model assumptions, especially with regard to the spatial and temporal scales of analysis, and computational strategies

- **The space-time mean structure**

  Considering only linear models, and covariates indexed in space (e.g., elevation, or distance to nearest road), in time (perhaps an area-wide seasonal effect or meteorological variable), or in space *x* time (including spatially resolved meteorological factors), the possibilities include (but are not limited to)

Spatio-Temporal Modeling

- *Separable* or *Additive* models in space and time,
  $\mu(\mathbf{s}, t) = X(\mathbf{s})\beta^s + X(t)\beta^t$,
- *Non-separable* functions of $X(\mathbf{s})$ and $X(t)$, which may be specified in terms of coefficients β^s of spatial covariates $X(\mathbf{s})$ varying in time, and/or coefficients β^t of temporal covariates $X(t)$ varying in space (random coefficient models).
- and *non-separable* mean structure in terms of spatiotemporal covariates, $X(\mathbf{s}, t)$, perhaps
  $\mu(\mathbf{s}, t) = X(\mathbf{s})\beta^{\mathbf{s}} + X(t)\beta^{\mathbf{t}} + X(\mathbf{s}, t)\beta^{\mathbf{st}}$.

Spatio-Temporal Modeling

Properties of space-time covariance functions

For all $(\mathbf{s}_1, t_1)$ and $(\mathbf{s}_2, t_2) \in R^d \times R$,

1. *Separable* Covariance Function

$$\{Y(\mathbf{s}_1, t_1), Y(\mathbf{s}_2, t_2)\} = C_S(\mathbf{s}_1, \mathbf{s}_2) \cdot C_T(t_1, t_2)$$

2. *Fully Symmetric* Covariance Function

$$\{Y({}_1, t_1), Y({}_2, t_2)\} = \{Y({}_1, t_2), Y({}_2, t_1)\}$$

3. *Stationary* Covariance Function

$$\{Z({}_1, t_1), Z({}_2, t_2)\} = ({}_1 - {}_2, t_1 - t_2)$$

Spatio-Temporal Modeling

Recall...

1. The class of stationary covariance functions is identical to the class of *symmetric positive definite* functions.

2. Products, sums, and convex combinations of positive definite functions are positive definite.

Spatio-Temporal Modeling

- Typically, parametric classes of stationary space-time covariance functions are constructed as sums or products of

1. Continuous space-time covariance functions

   $\rightarrow$ see the following slides

2. Space-time *nugget effect*

$$C(\boldsymbol{h},u) = a\,\delta_{(\boldsymbol{h},u)=(\mathbf{0},0)} + b\,\delta_{\boldsymbol{h}=\mathbf{0}} + c\,\delta_{u=0}, \;\; (\boldsymbol{h},u) \in \mathbb{R}^d \times \mathbb{R}$$

   where a, b, and c are nonnegative constants

Spatio-Temporal Modeling

- **Separable (continuous) covariance functions**

$$C(\boldsymbol{h}, u) = C_S(\boldsymbol{h}) \cdot C_T(u), \ (\boldsymbol{h}, u) \in \mathbb{R}^d \times \mathbb{R}$$

For example, take

$$C_S(\boldsymbol{h}) = c_S(||\boldsymbol{h}||)$$

and

$$C_T(u) = c_T(|u|),$$

where $c_S(\cdot)$ and $c_T(\cdot)$ are (possibly distinct) parametric classes of isotropic covariance function such as

→ Matérn

→ Powered exponential

Spatio-Temporal Modeling

- **Non-separable (continuous) covariance functions**

Bochner's Theorem

Suppose that C is a continuous and symmetric function on $\mathbb{R}^d \times \mathbb{R}$. Then C is a covariance function if and only if it is of the form

$$C(\boldsymbol{h}, u) = \int\int e^{i(\boldsymbol{h}'\boldsymbol{\omega} + u\tau)} dF(\omega, \tau), \ (\boldsymbol{h}, u) \in \mathbb{R}^d \times \mathbb{R},$$

where F is a finite, non-negative, and symmetric measure on $\mathbb{R}^d \times \mathbb{R}$.

If C is *integrable*, the spectral density f corresponding to F exists and Bochner's Theorem reduces to

$$C(\boldsymbol{h}, u) = \int\int e^{i(\boldsymbol{h}'\boldsymbol{\omega} + u\tau)} f(\omega, \tau) d\omega d\tau, \ (\boldsymbol{h}, u) \in \mathbb{R}^d \times \mathbb{R}$$

Spatio-Temporal Modeling

1. Cressie and Huang (1999)

   Theorem

   Suppose that C is a continuous, bounded, integreble, and symmetric function on $\mathbb{R}^d \times \mathbb{R}$. Then C is a stationary covariance if and only if

   $$\rho(\omega, u) = \int e^{-i\boldsymbol{h}'\omega} C(\boldsymbol{h}, u) d\boldsymbol{h}, \; u \in \mathbb{R},$$

   is positive definite for almost all $\omega \in \mathbb{R}^d$

   $$C(\boldsymbol{h}, u) = \int e^{i\boldsymbol{h}'\boldsymbol{\omega}} \rho(\boldsymbol{\omega}, u) d\boldsymbol{\omega}, \; (\boldsymbol{h}, u) \in \mathbb{R}^d \times \mathbb{R}$$

   where $\rho(\boldsymbol{\omega}, u)$, $u \in \mathbb{R}$, is a continuous positive definite function for all $\boldsymbol{\omega} \in \mathbb{R}^d$.

   → requires closed-form Fourier inversion in $\mathbb{R}^d$.

Spatio-Temporal Modeling

2. Gneiting (2002)

$$C(\boldsymbol{h}, u) = \frac{1}{\psi(u^2)^{d/2}} \varphi\left(\frac{||\boldsymbol{h}||^2}{\psi(u^2)}\right), \ (\boldsymbol{h}, u) \in \mathbb{R}^d \times \mathbb{R}$$

where $\varphi(r)$, $r \geq 0$, is a *completely monotone* function and $\psi(r)$, $r \geq 0$, is a positive function with a *completely monotone* derivative

Spatio-Temporal Modeling

Example:

If $\varphi(r) = \sigma^2 \exp(-cr^\gamma)$ and $\psi(r) = (1 + ar^\alpha)^\beta$, then

$$C(\boldsymbol{h}, u) = \frac{\sigma^2}{(1 + a|u|^{2\alpha})^{\beta d/2}} \exp\left(-\frac{c||\boldsymbol{h}||^{2\gamma}}{(1 + a|u|^{2\alpha})^{\beta\gamma}}\right)$$

→ a and c are nonnegative scale parameters of time and space, respectively

→ α and $\gamma \in (0, 1]$ are *smoothness* parameters

→ $\beta \in (0, 1]$ is a *space-time interaction* parameter

→ σ^2 is the *variance* of the space-time process

Spatio-Temporal Modeling

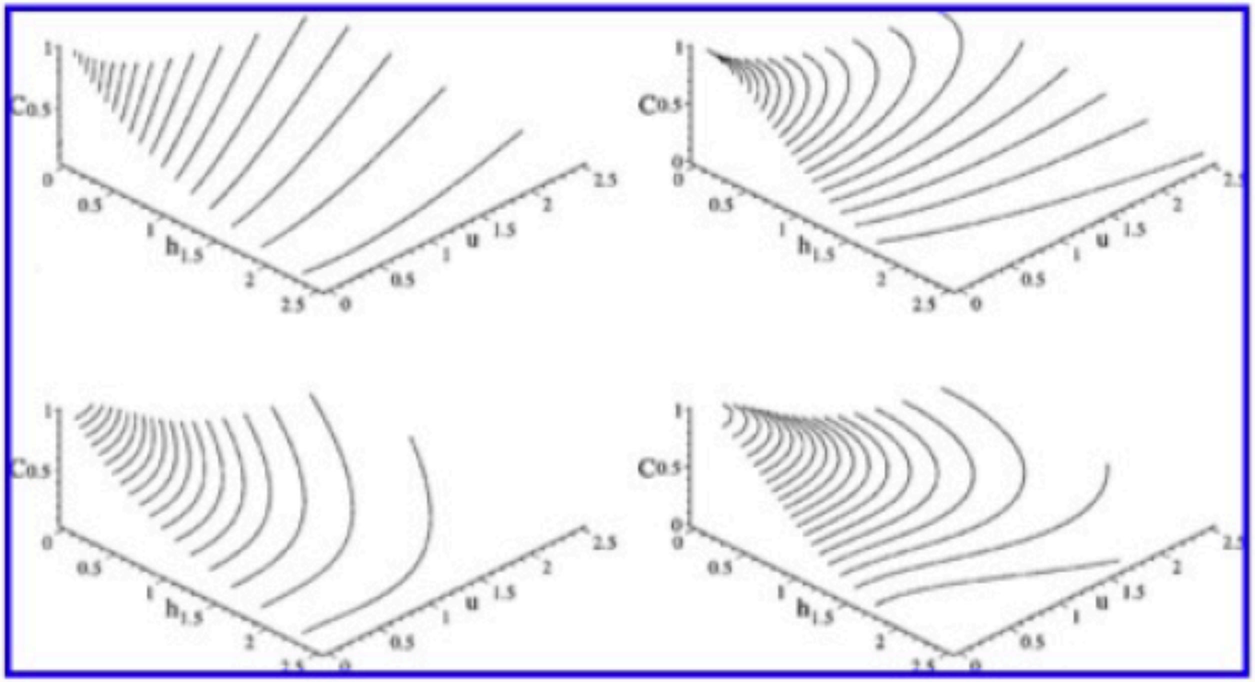


Figure 1. Contour Plots of the Space–Time Covariance Function (13) Versus the Modulus of the Spatial Lag, $\|\mathbf{h}\|$, and the Temporal Lag, $|u|$. The functions attain their maximum, $C(\mathbf{0};0)=1$, at the origin, and the contour lines are equidistant at .95, .9, . . . , .05. Upper left: $\alpha = 1/2$, $\gamma = 1/2$. Upper right: $\alpha = 1/2$, $\gamma = 1$. Lower left: $\alpha = 1$, $\gamma = 1/2$. Lower right: $\alpha = 1$, $\gamma = 1$.

(Gneiting, 2002)

Spatio-Temporal Modeling

3. Mixtures of separable covariance functions

Theorem

Let μ be a finite, nonnegative measure on a non-empty set Θ. Suppose that for each $\theta \in \Theta$, C_S^θ and C_T^θ are stationary covariances on $\mathbb{R}^d$ and $\mathbb{R}$, respectively, and suppose that $C_S^\theta(\mathbf{0})C_T^\theta(0)$ has finite integral over Θ. Then

$$C(\boldsymbol{h}, u) = \int C_S^\theta(\boldsymbol{h})C_T^\theta(u)d\mu(\theta), \ (\boldsymbol{h}, u) \in \mathbb{R}^d \times \mathbb{R}$$

is a stationary covariance function on $\mathbb{R}^d \times \mathbb{R}$

Examples:

- De Iaco *et al.* (2001)

$$C(\boldsymbol{h}, u) = a_0 C_S^0(\boldsymbol{h})C_T^0(u) + a_1 C_S^1(\boldsymbol{h}) + a_2 C_T^2(u), \ (\boldsymbol{h}, u) \in \mathbb{R}^d \times \mathbb{R}$$

where a_0, a_1, and a_2 are nonnegative coefficients

Spatio-Temporal Modeling

- De Iaco *et al.* (2002)

  → μ is a gamma distribution on $\Theta = [0, \infty)$

  → both C_S^θ and C_T^θ are of powered exponential type

$$\Rightarrow C(\boldsymbol{h}, u) = \sigma^2 \left(1 + \left\|\frac{\boldsymbol{h}}{a}\right\|^\alpha + \left|\frac{u}{b}\right|^\beta\right)^{-\gamma}, \ (\boldsymbol{h}, u) \in \mathbb{R}^d \times \mathbb{R}$$

where $\alpha \in (0, 2]$, $\beta \in (0, 2]$, $\gamma > 0$, $a > 0$, $b > 0$, and $\sigma > 0$

Spatio-Temporal Modeling

- **Non-fully symmetric (continuous) covariance functions**
  - Lagrangian reference frame approach
    [Cox and Isham, 1988; Ma, 2003; Gneiting *et al.*, 2007]

    $$C(\boldsymbol{h}, u) = \mathrm{E}[C_S(\boldsymbol{h} - \boldsymbol{V} u)], \; (\boldsymbol{h}, u) \in \mathbb{R}^d \times \mathbb{R}$$

    where $\boldsymbol{V} \in \mathbb{R}^d$ is a random velocity vector describing the time-forward movement of the spatial field

    $\rightarrow$ *Special Case:* $\boldsymbol{V} = v$, where v is constant (*frozen field model*)

  - Diffusion equations, SPDE approaches
    [Jun and Stein, 2004; Stein, 2005; and others]

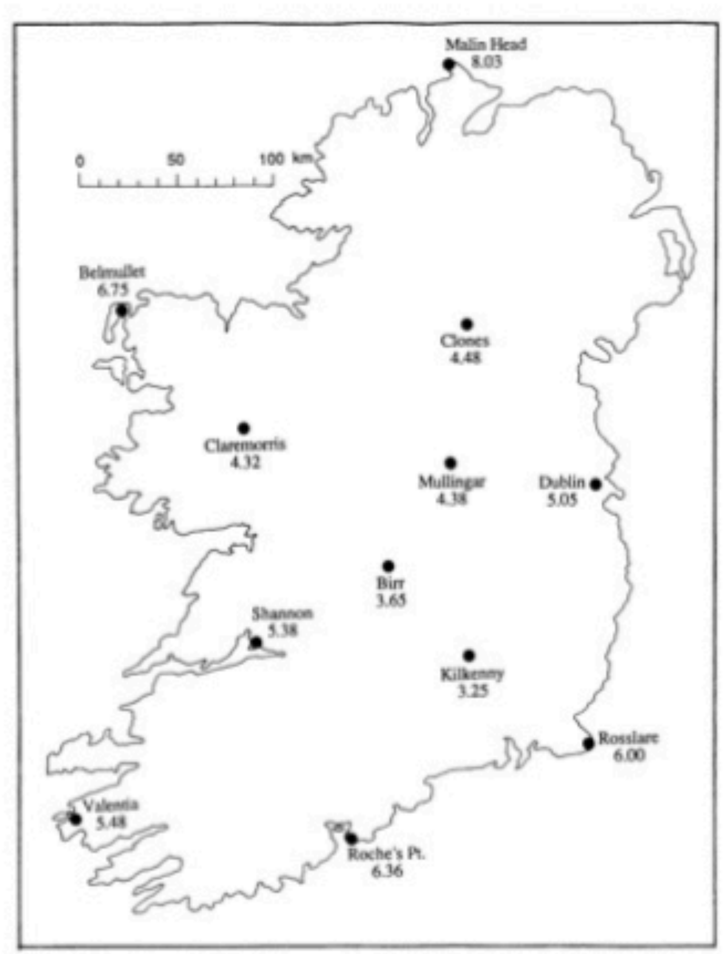
Lecture 8a: Spatio-Temporal Processes

IRISH WIND DATA EXAMPLE

[Haslett and Raftery, 1989]

- Daily average wind speed at 11 meteorological stations
- Training period: 1961-1970
  Test period: 1971-1978
- Data transformations:
  1. take square root
  2. fit and remove a common seasonal component
  3. remove station-specific means

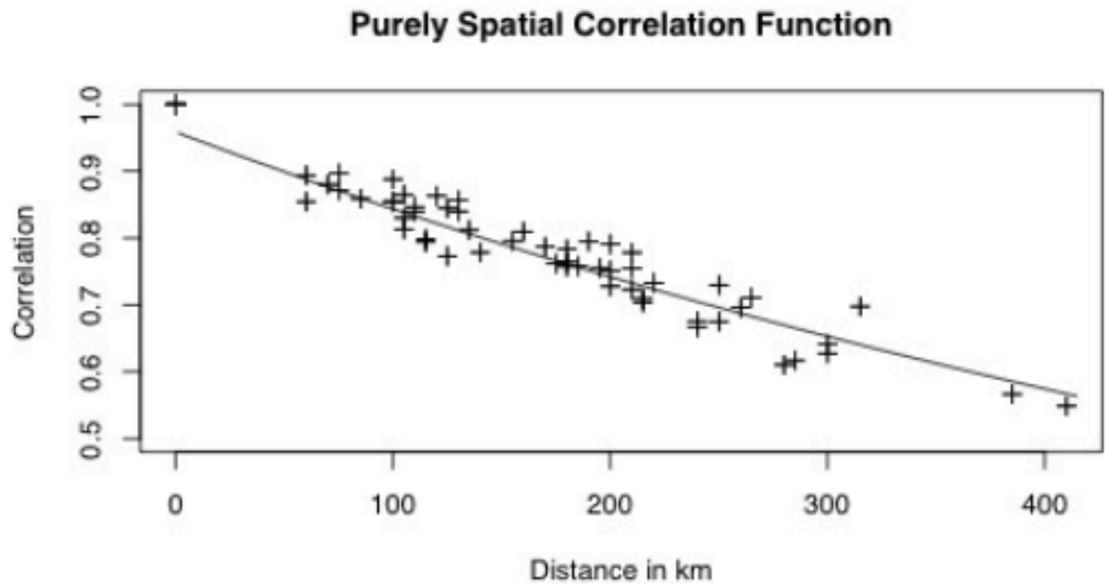
  $\Rightarrow$ *velocity measures*



(Haslett and Raftery, 1989)

Lecture 8a: Spatio-Temporal Processes

- **Exploratory data analysis** [Gneiting *et al.*, 2007]
  - Purely spatial and purely temporal empirical *correlations*



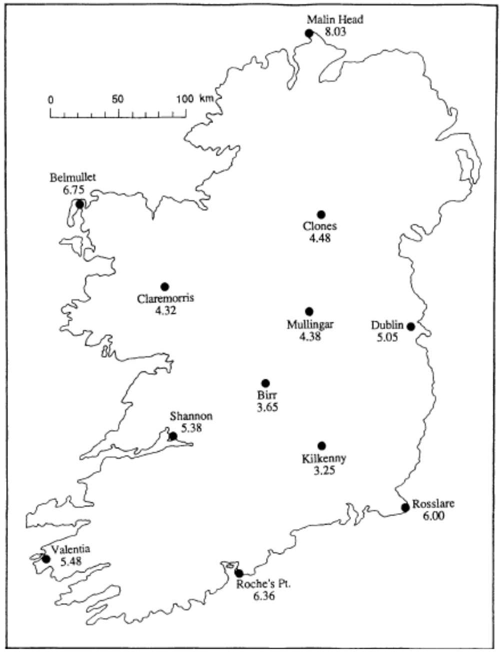
(Gneiting *et al.*, 2007)

$C_S(\boldsymbol{h}) = (1-\nu)\exp(-c||h||) + \nu\delta_{\boldsymbol{h}=\boldsymbol{0}}$

$\hat{\nu} = 0.0415$ and $\hat{c} = 0.00128$

$C_T(u) = (1+a|u|^{2\alpha})^{-1}$, for $|u| \leq 3$

$\hat{a} = 0.973$ and $\hat{\alpha} = 0.834$



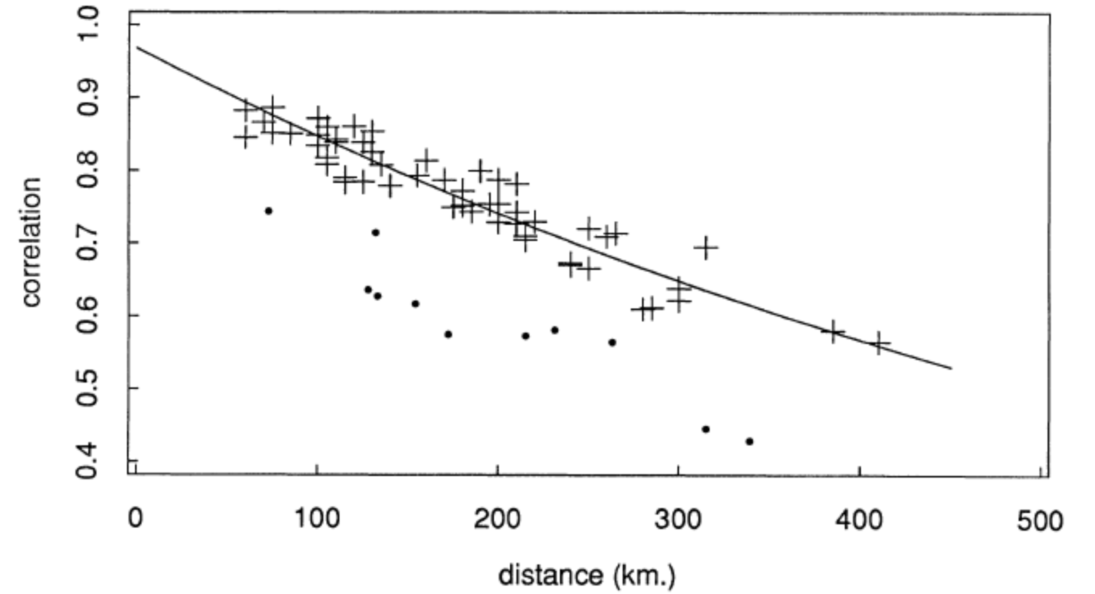
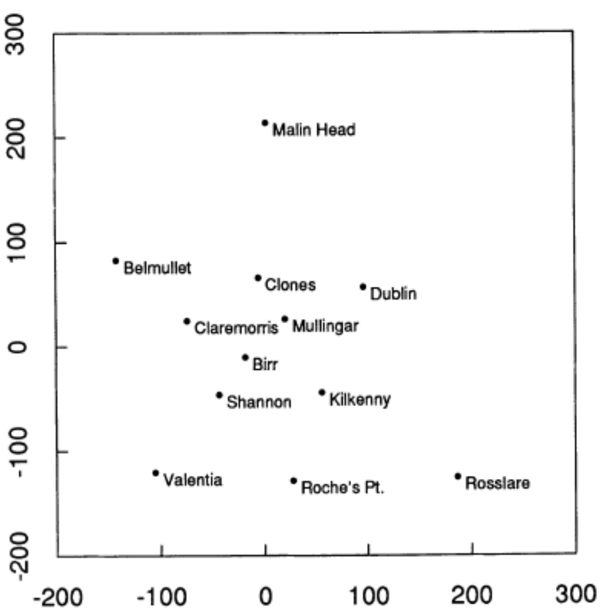
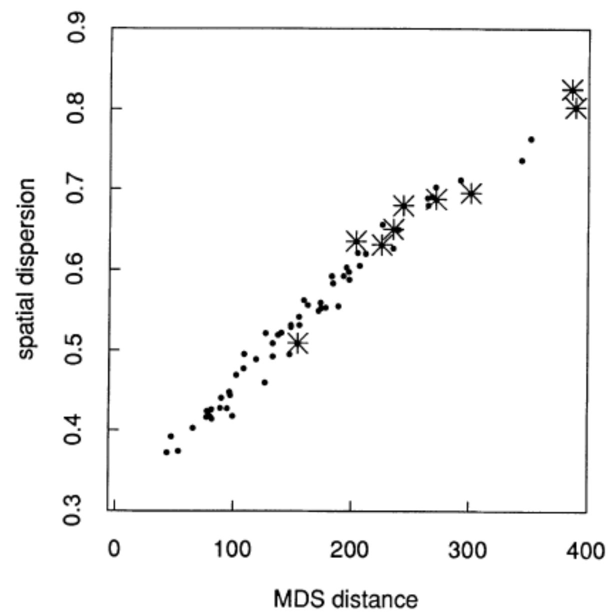


Fig. 3. Distance–correlation plot. Each cross corresponds to a pair of synoptic stations; the dots correspond to pairs which include Rosslare. The full line is the fitted relationship (3.3) with $\alpha = 0.968$ and $\beta = 0.00134$.

DISCUSSION OF THE PAPER BY HASLETT AND RAFTERY



G&S 1989 Deformation model analysis



MDS representation of the IMS monitoring stations based on estimated spatial dispersions ι

Fig. 14. Plot of spatial dispersion v_{ij} versus interstation distance in the MDS representation: $*$, station pair involving Rosslare

Lecture 8a: Spatio-Temporal Processes

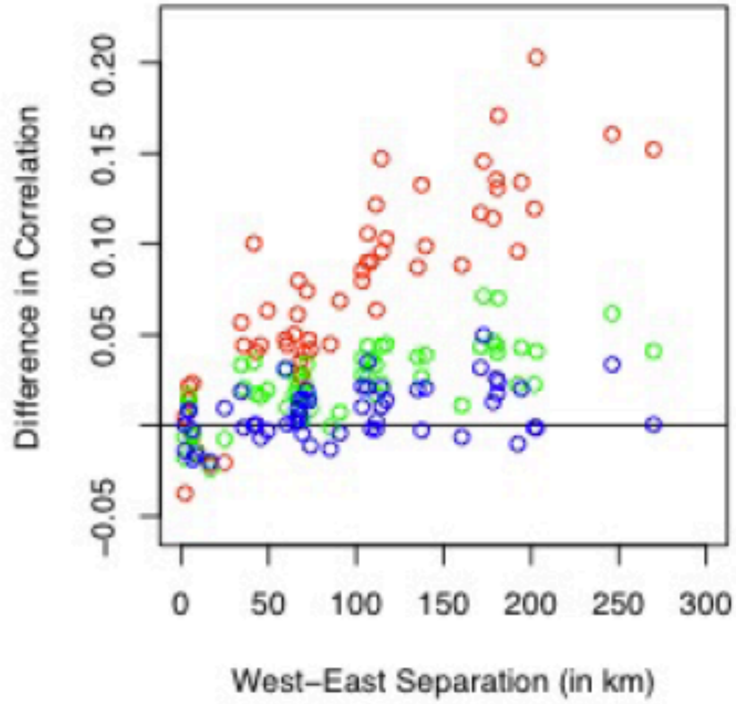
- Directional empirical correlations

Westerly Station	Easterly Station	WE	EW
Valentia	Roche's Point	.48	.35
Belmullet	Clones	.52	.39
Claremorris	Mullingar	.51	.41
Claremorris	Dublin	.50	.36
Shannon	Kilkenny	.51	.39
Mullingar	Dublin	.49	.45

(Gneiting *et al.*, 2007)

Lecture 8a: Spatio-Temporal Processes

- Differences in the west-to-east and east-to-west empirical correlations **Evidence of asymmetry**



(Gneiting *et al.*, 2007)

Temporal lag: one day (red), two day (green), three day (blue)

Lecture 8a: Spatio-Temporal Processes

- Stationary space-time models for the Irish wind data [Gneiting *et al.*, 2007]
  1. *Separable*

     $$C_{SEP}(\boldsymbol{h}, u) = C_S(\boldsymbol{h}) \cdot C_T(u)$$

     where, as before,

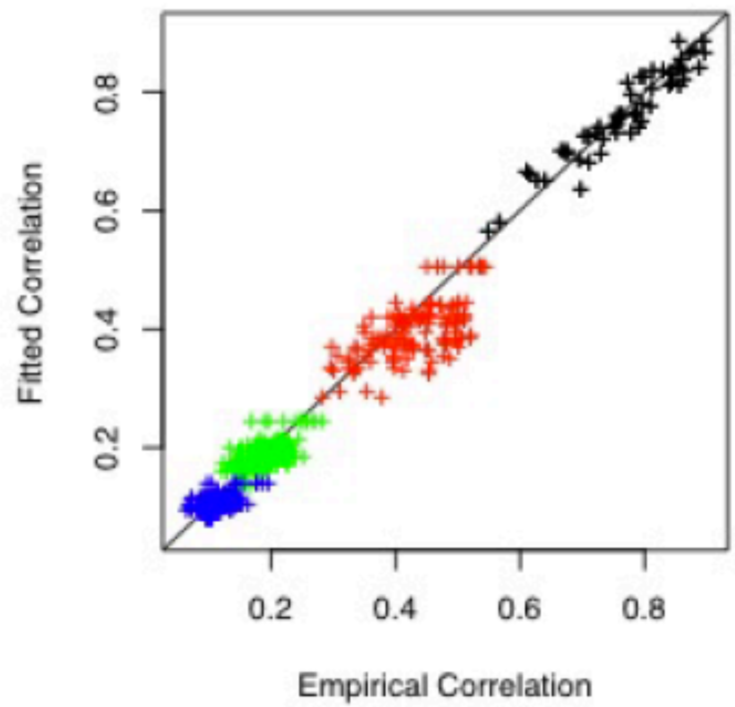
     $$C_S(\boldsymbol{h}) = (1 - \nu) \exp(-c||h||) + \nu \delta_{\boldsymbol{h}=\mathbf{0}}$$

     $$\textit{WLS Estimates: } \hat{\nu} = 0.0415 \text{ and } \hat{c} = 0.00128$$

     $$C_T(u) = (1 + a|u|^{2\alpha})^{-1}, \text{ for } |u| \leq 3$$

     $$\textit{WLS Estimates: } \hat{a} = 0.973 \text{ and } \hat{\alpha} = 0.834$$

Lecture 8a: Spatio-Temporal Processes



(Gneiting *et al.*, 2007)

Temporal lags: zero day (black), one day (red), two day (green), three day (blue)

Lecture 8a: Spatio-Temporal Processes

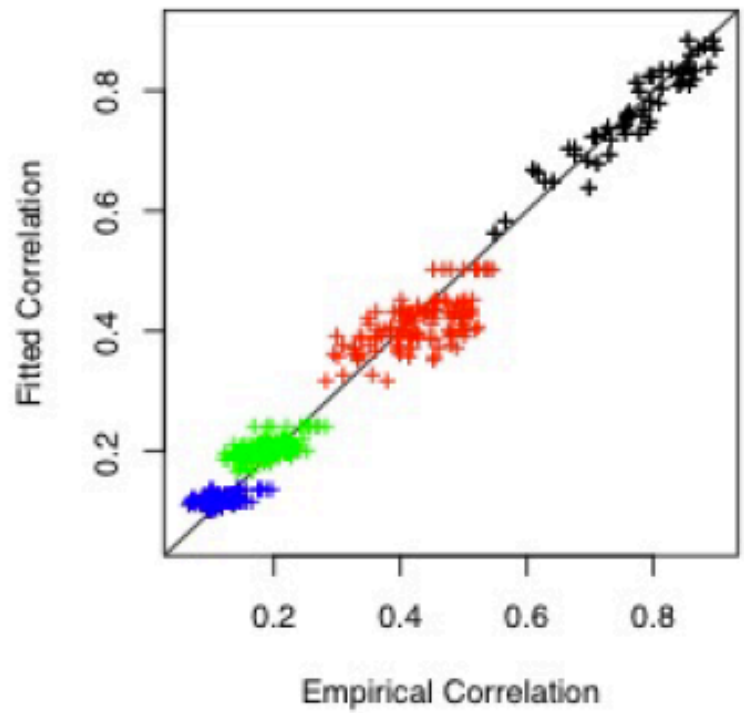
2. *Fully symmetric, non-separable*

$$C_{FS}(\boldsymbol{h}, u) = \frac{1-\nu}{1+a|u|^{2\alpha}} \left(\exp\left(-\frac{c||\boldsymbol{h}||}{(1+a|u|^{2\alpha})^{\beta/2}} \right) + \frac{\nu}{1-\nu} \delta_{\boldsymbol{h}=\mathbf{0}} \right)$$

where $\beta \in [0,1]$ is the *space-time interaction* parameter.

WLS estimate: $\hat{\beta} = 0.681$

Lecture 8a: Spatio-Temporal Processes



(Gneiting *et al.*, 2007)

Temporal lags: zero day (black), one day (red), two day (green), three day (blue)

Lecture 8a: Spatio-Temporal Processes

3. *General (not fully symmetric, non-separable)*

$$C_{STAT}(\boldsymbol{h}, u) = (1-\lambda)C_{FS}(\boldsymbol{h}, u) + \lambda C_{LGR}(\boldsymbol{h}, u)$$

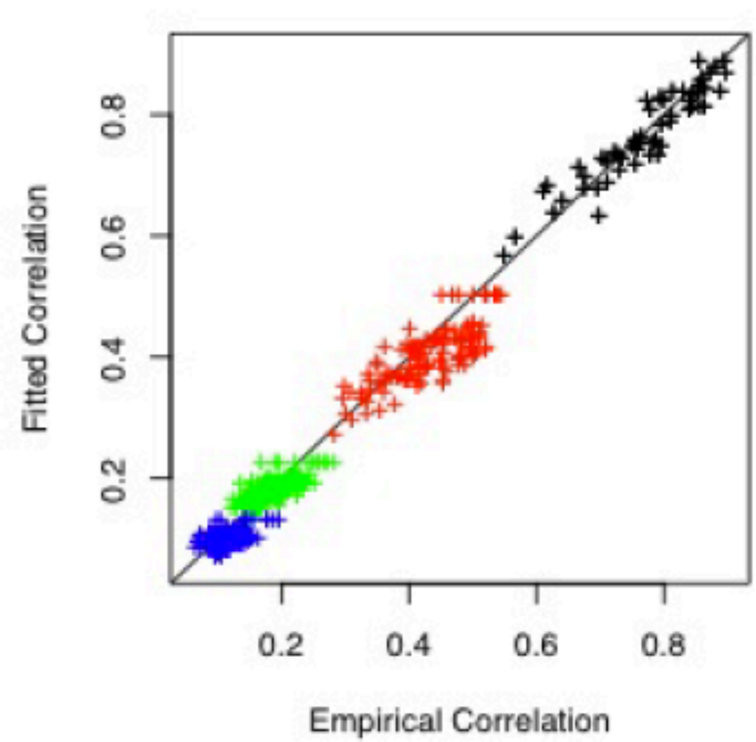
where $\boldsymbol{h} = (h_1, h_2)'$ and

$$C_{LGR}(\boldsymbol{h}, u) = \left(1 - \frac{1}{2\nu}|h_1 - vu|\right)_+$$

(The parameter $v \in \mathbb{R}$ can be interpreted as the *longitudinal velocity*.)

WLS estimate: $\hat{v} = 234km/d = 2.71m/s$

Lecture 8a: Spatio-Temporal Processes



(Gneiting *et al.*, 2007)

Temporal lags: zero day (black), one day (red),
two day (green), three day (blue)