Conditional Modelling of Extreme Wind Gusts by Bivariate Brown-Resnick Processes

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joint work with
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Recall the talks by D. Cooley and M. Ribatet

Extreme value theory

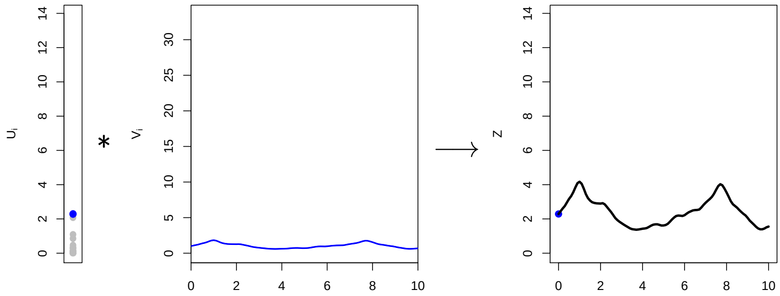
- Generalized Extreme Value (GEV) Distribution
  - limit law of i.i.d. maxima
  - annual maxima
- Generalized Pareto Distriubution
  - exceedances overhigh thresholds
  - tail equivalent to GEV
- Max-stable processes
  - spatial concept of extremes
  - statistical inference

Outline

Spectral Representation (de Haan, 1984)

- $K \subset \mathbb{R}^d$ compact
- $\mathcal{H}$: space of spectral functions $K \to [0, \infty)$ with measure H
- $\Pi = \sum \delta_{(U_i, V_i)}$: Poisson point process on $(0, \infty) \times \mathcal{H}$ with intensity $u^{-2}\,\mathrm{d}u \cdot H(\mathrm{d}f)$

$$X(t) = \max_{i \in \mathcal{N}} U_i V_i(t), \quad t \in K$$



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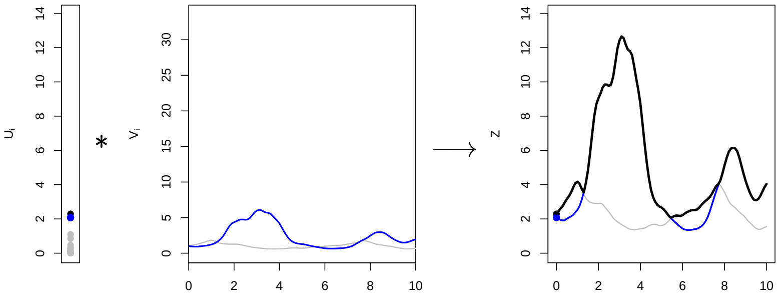

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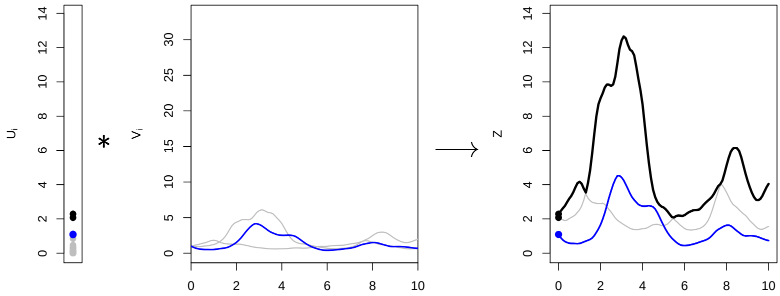

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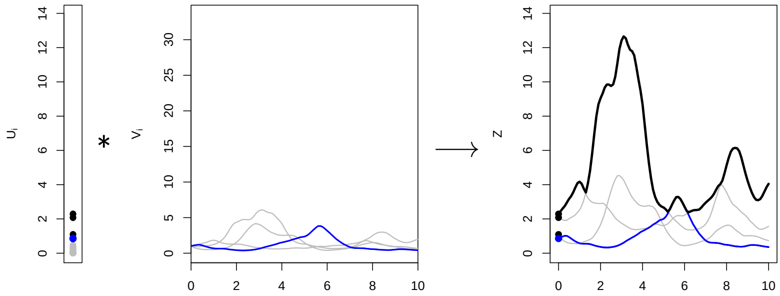

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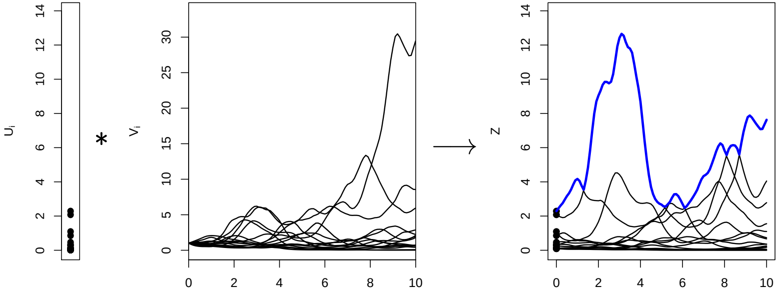
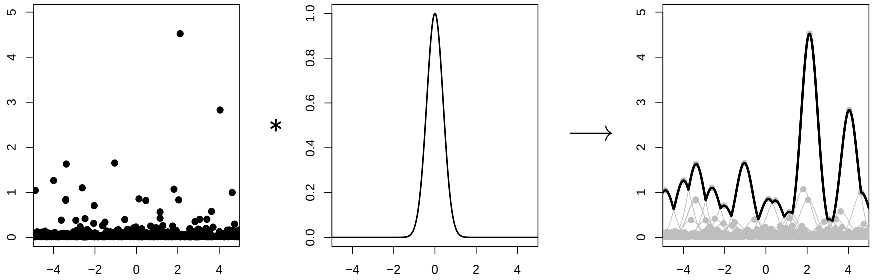


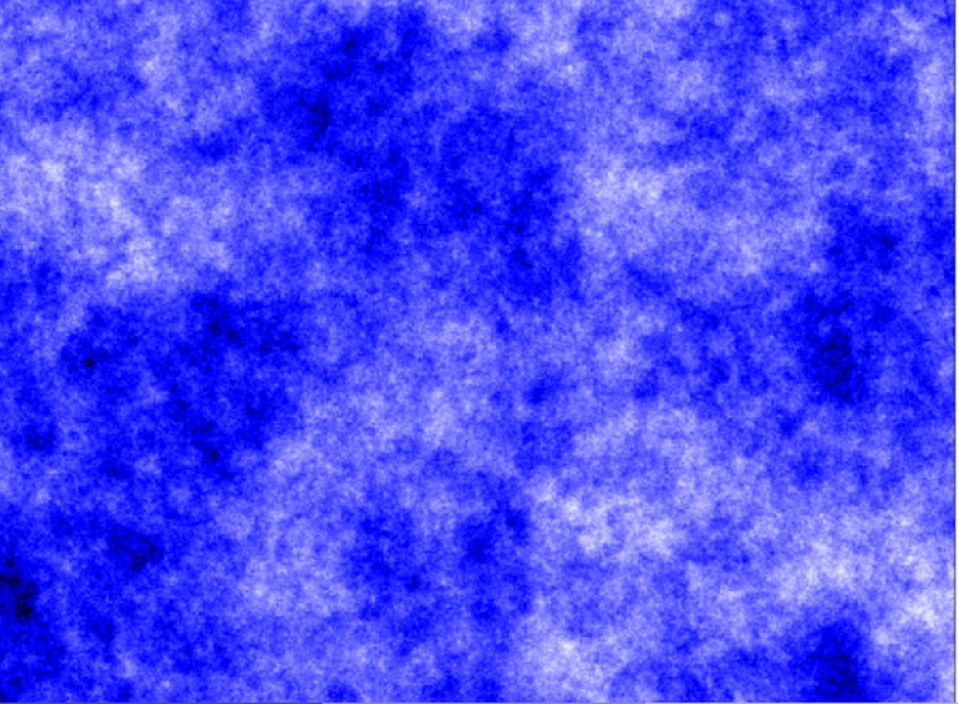
Moving Maxima (e.g. Smith Process, 1990)

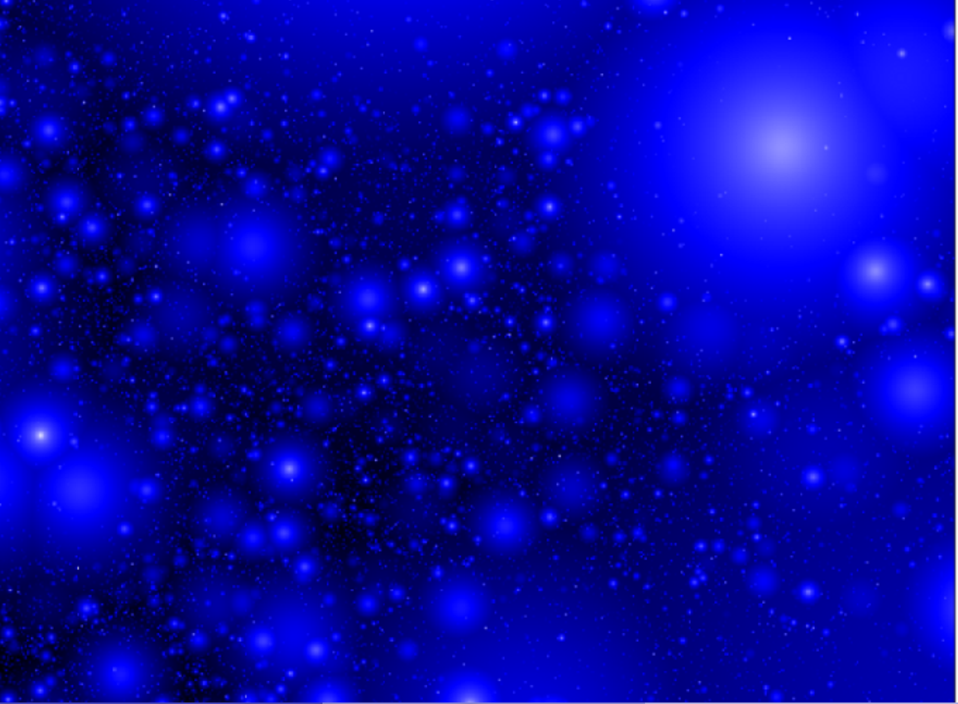
- $\sum_{i \in \mathcal{N}} \delta_{(U_i, S_i)}$: Poisson point process on $(0, \infty) \times \mathbb{R}^d$ with intensity $u^{-2}\mathrm{d}u \times \mathrm{d}s$
- F: deterministic "shape function"

$$X(t) = \max_{i \in \mathcal{N}} (U_i \cdot F(t - S_i))$$

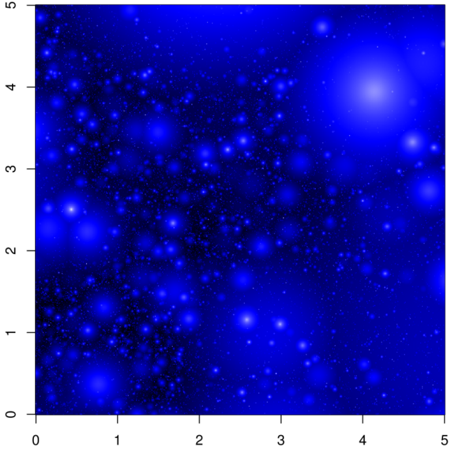


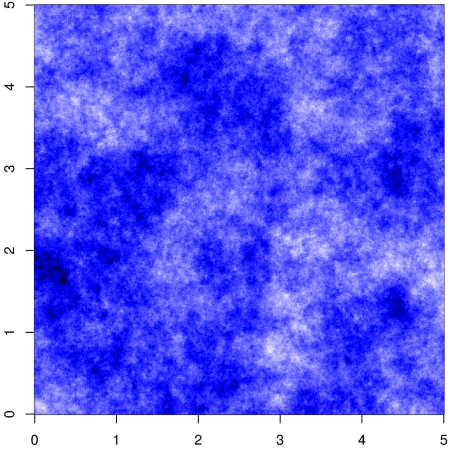
$\rightsquigarrow$ spectral functions are shifted shape functions $F(\cdot - S_i)$





What do these pictures have in common?





Tail correlation function (TCF)

$X = \{X(t)\}_{t\in\mathbb{R}^d}$: a stationary stochastic process

$$\chi(t) := \lim_{x\to x^*} \mathbb{P}(X_t > x \,|\, X_o > x), \qquad t \in \mathbb{R}^d.$$

(provided limits exist; $x^* =$ upper endpoint)

Comments

- correlation function for tail dependence
- invariant under continuous isotonic marginal transformations
- different names in the literature:

(upper) tail dependence coefficient	[Beirlant *et al.* '04, Davis/Mikosch '09, Falk '05]
χ*-measure*	[Beirlant *et al.* '04, Coles *et al.* '99]
extremal coefficient function	[Fasen *et al.* '10]
. . .	

- estimable by F-madogram (Cooley, Naveau & Poncet, 2006)

Properties

$$\chi(t) = \lim_{x \to x^*} \mathbb{P}(X_t > x \,|\, X_o > x), \qquad t \in \mathbb{R}^d$$

- χ is positive semidefinite (a non-negative correlation function):

$$\sum_{i=1}^n \sum_{j=1}^n a_i \chi(t_i - t_j) a_j \geq 0, \qquad \begin{array}{l} \forall\,(t_1, \ldots, t_n) \in (\mathbb{R}^d)^n, \\ \forall\,(a_1, \ldots, a_n) \in \mathbb{R}^n. \end{array}$$

- χ satisfies further inequalities:

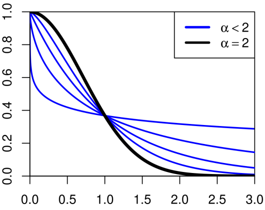
$$\begin{array}{rl} \chi_{ij} & \geq 0 \\ \chi_{ij} + \chi_{ik} - \chi_{jk} & \leq 1 \\ (\chi_{ij} + \chi_{ik} + \chi_{il}) - (\chi_{jk} + \chi_{jl} + \chi_{kl}) & \leq 1 \end{array} \qquad \begin{array}{l} \forall\,(t_i, t_j, t_k, t_l) \in (\mathbb{R}^d)^4 \\ \text{with } \chi_{ij} = \chi(t_i - t_j). \end{array}$$

[Davis/Mikosch '09, S./Tawn '03, Strokorb '13]

Example: Parametric families

Powered Exponential

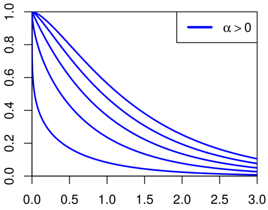
$$\rho_\alpha(r) = \exp(-r^\alpha)$$



CF for $\alpha \in (0, 2]$

Whittle-Matérn

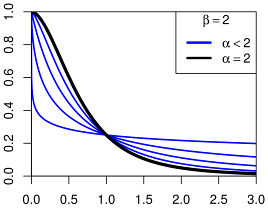
$$\rho_\alpha(r) = \frac{2^{1-\alpha}}{\Gamma(\alpha)} r^\alpha K_\alpha(r)$$



CF for $\alpha \in (0, \infty)$

Cauchy

$$\rho_{\alpha,\beta}(r) = (1 + r^\alpha)^{-\beta}$$



CF for $\alpha \in (0, 2]$
(for all $\beta > 0$)

Example: Parametric families

Powered Exponential	**Whittle-Matérn**	**Cauchy**
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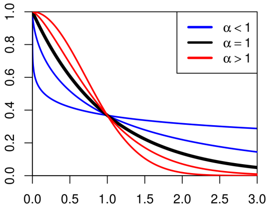

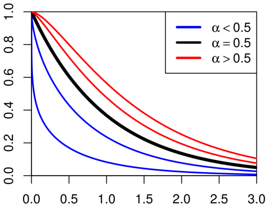

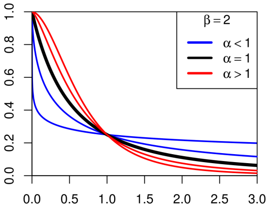

CF for $\alpha \in (0, 2]$	CF for $\alpha \in (0, \infty)$	CF for $\alpha \in (0, 2]$ (for all $\beta > 0$)
TCF for $\alpha \in (0, 1]$	TCF for $\alpha \in (0, 0.5]$	TCF for $\alpha \in (0, 1]$ (for all $\beta > 0$)

Background picture

Construction

- Moving maxima process:

$$X(t) = \max_{(u,s)\in\Pi} uF(t-s)$$

  Π a Poisson point process with intensity $u^{-2}\,\mathrm{d}u\,\mathrm{d}s$
- deterministic shape function F in $\mathbb{R}^3$,

$$F(t) = \frac{1+4\|t\|}{\pi^{3/2}\|2t\|^{5/2}} e^{-2\|t\|}$$

- background picture is the 2-dimensional cross-section of a 3-dimensional realisation of $log(X)$

Background picture, part II

Properties

- tail correlation function equals

$$\chi(h) = \lim_{x \to x^*} \mathbb{P}(X(h) > x \mid X(0) > x) = \mathrm{erfc}(\sqrt{\|h\|})$$

where $\mathrm{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-y^2} \, \mathrm{d}y$

Background picture, part II

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I.e., identical to the (classical) Brown-Resnick process.

Background picture, part II

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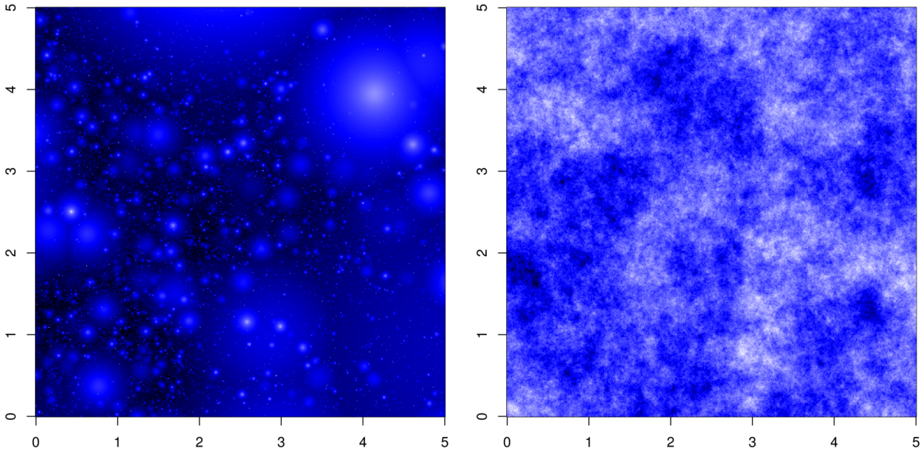
  I.e., identical to the (classical) Brown-Resnick process.
- discontinuous everywhere

So, these pictures have the tail correlation function in common!



Numerical wheather forecast

System of six partial differential equations

- equations include conservation of momentum, mass, energy and entropy, and equation of state
- two velocity components, density, pressure, temperature, humidity

Deterministic forecasts of future states of the atmosphere

- discretization
- run forward in time

Initial conditions

- data assimilation systems
- describing current state of the atmosphere on a 3d grid

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Tim Palmer (2000):

Although forecasters have traditionally viewed weather prediction as deterministic, a culture change towards probabilistic forecasting is in progress.

Postprocessing

NWP ensembles are subject to model biases and typically they show a lack of calibration

Univariate postprocessing

- regression based approach (Gneiting *et al.*, 2005) using normal distribution assumptions
- Baysian approach (Raftery *et al.*, 2005)

Multivariate and spatial postprocessing

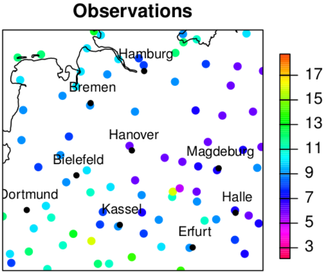
- empirical copula coupling (Schuhen *et al.*, 2012)
- score functions

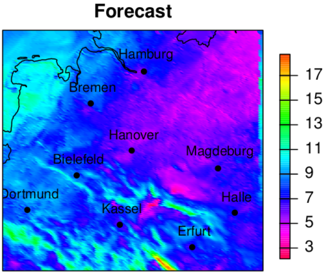
Postprocessing of extremes

- ansatz with normal distribution should be replaced by GEV
- empirical copula coupling coupling might be replaced by a spatial model
- quality control essential
- spatial model, hence downscaling, might be worthwhile

Extreme Wind Gusts

- wind gusts are strongly varying in space
- high uncertainty in forecasts, particularly for extreme wind gusts





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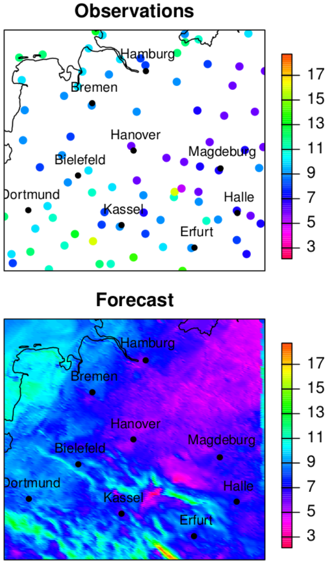

Goal:

Model for the (observed) wind gusts $V_{\max}^{\mathrm{obs}}$ conditional on the forecast $V_{\max}^{\mathrm{pred}}$

Model for Extreme Wind Gusts

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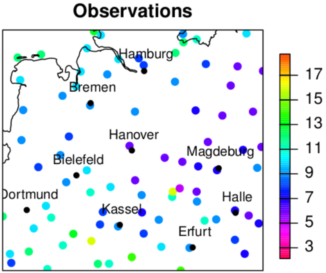
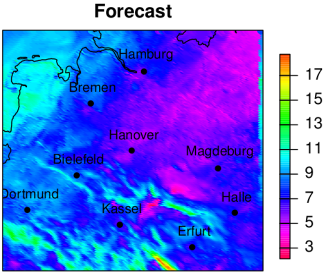
Model for the (observed) wind gusts $V_{\max}^{\text{obs}}$ conditional on the forecast $V_{\max}^{\text{pred}}$

Two Modelling Steps:

1. model for marginal distributions (at single location) of $V_{\max}^{\text{obs}}$ & $V_{\max}^{\text{pred}}$
2. model for spatial dependence & dependence between observation and prediction
   $\rightsquigarrow$ bivariate stochastic process



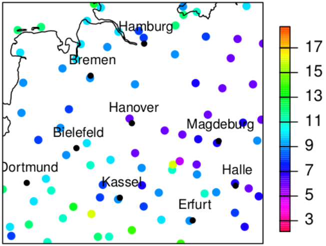


The Data

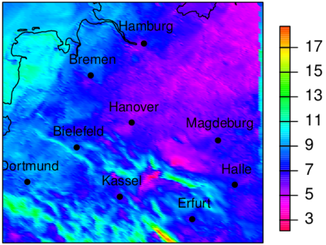
Observation data:
for the maximal wind speed $\rightsquigarrow V_{\max}^{\mathrm{obs}}$

- at 116 DWD stations in Northern Germany
- for 358 days (03/2011 – 02/2012)

Observations



Forecast



The Data

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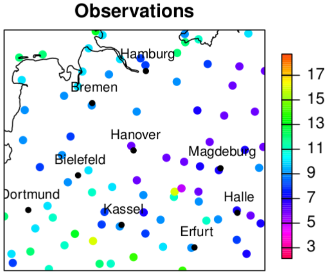
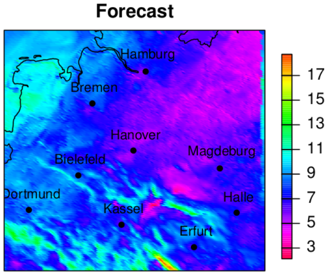
Forecast data:
from COSMO-DE EPS

- on a grid with mesh size 2.8 km covering Germany
- 20 ensemble members

1. for the maximal wind speed $\rightsquigarrow V_{\max}^{\text{pred}}$
2. for the mean wind speed





Marginal Model for Wind Speed

(single) **wind speed** $V(l,d)$ at location l and day d:

$$V(l,d) =_d \sqrt{\mathrm{Var}\{V(l,d)\}}V_0 + \mathbb{E}\{V(l,d)\}$$

with V_0 following some standardized distribution

"weather parameters" $\mathbb{E}\{V(l,d)\}$ and $\mathrm{Var}\{V(l,d)\}$:

- reflect the general weather situation
- contain seasonal effects
- are assumed to be "known" to the forecaster

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$\rightsquigarrow$ can be estimated from mean wind ensemble forecast

Marginal Model for Extreme Wind Gusts

(single) **wind speed** $V(l,d)$ at location l and day d:

$$V(l,d) =_d \sqrt{\mathrm{Var}\{V(l,d)\}}V_0 + \mathbb{E}\{V(l,d)\}$$

with V_0 following some standardized distribution

observed **single** wind speed: 3-second average
observed **maximal** wind speed: highest 3-second average per day
$\rightsquigarrow$ GEV

maximal wind speed $V_{\max}(l,d)$ at location l and day d:

$$\mathbb{P}\left(\frac{V_{\max}(l,d) - \mathbb{E}\{V(l,d)\}}{\sqrt{\mathrm{Var}\{V(l,d)\}}} \le x\right) \approx \exp\left(-\left(1+\xi\frac{x-\mu}{\sigma}\right)_+^{-1/\xi}\right)$$
$$= G_{\xi,\mu,\sigma}(x)$$

Marginal Model for Extreme Wind Gusts (cont'd)

Observations

$$\frac{V_{\mathsf{max}}^{\mathrm{obs}}(l,d) - \mathbb{E}\{V(l,d)\}}{\sqrt{\mathrm{Var}\{V(l,d)\}}} \sim GEV(\xi^{\mathrm{obs}}, \mu^{\mathrm{obs}}, \sigma^{\mathrm{obs}})$$

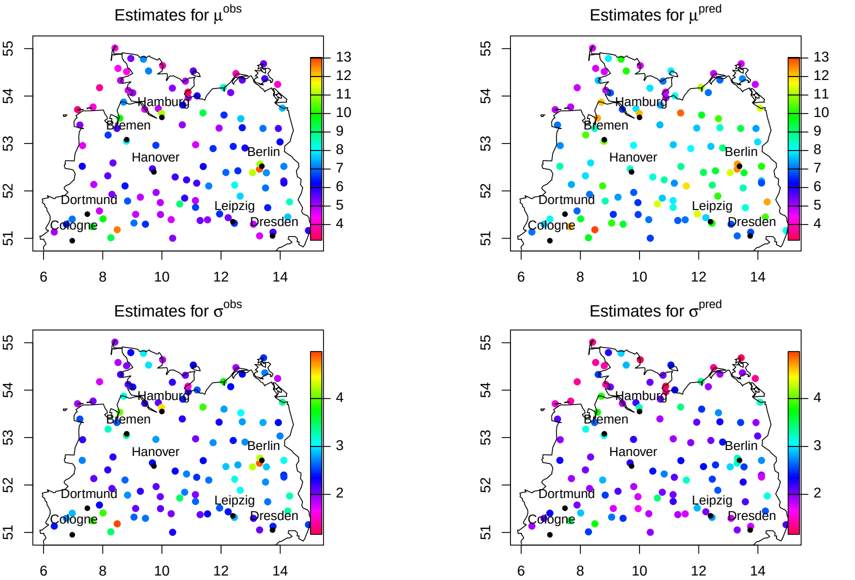
Forecast

$$\frac{V_{\mathsf{max}}^{\mathrm{pred}}(l,d) - \mathbb{E}\{V(l,d)\}}{\sqrt{\mathrm{Var}\{V(l,d)\}}} \sim GEV(\xi^{\mathrm{pred}}, \mu^{\mathrm{pred}}, \sigma^{\mathrm{pred}})$$

GEV parameters (ξ, μ, σ):

- ξ constant in space in time
- error model allows μ, σ to vary spatially
- estimated via maximum likelihood

GEV Parameters



$\rightarrow$ necessity to model spatial prediction and spatial observation together

Intrinsically stationary processes

- univariate case:
  - $Y_s(t) = W^{(1)}(t+s) - W^{(1)}(t)$ is (weakly) stationary
  - variogramm γ,

$$\gamma(t,s) = \mathrm{Var}(W^{(1)}(t) - W^{(1)}(s))$$

depends only on the distance vector $t - s$

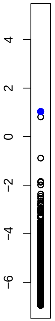
- multivariate set up:
  - different approaches
  - we need that the pseudo-variogram $\gamma(s,t) = (\gamma_{ij}(s,t))_{1 \leq i,j \leq 2}$ with

$$\gamma_{ij}(s,t) = Var(W^{(i)}(s) - W^{(j)}(t))$$

only depends on $s - t$.

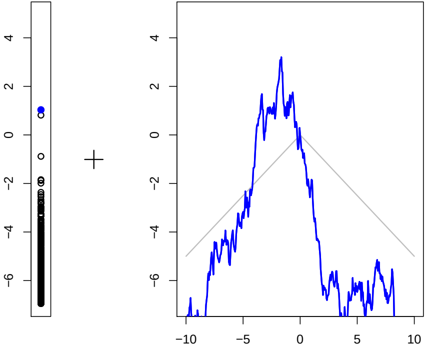
Point Process Construction (Brown & Resnick 1977)

- $\{U_k,\ k \in \mathcal{N}\}$: PPP on $\mathbb{R}$ with intensity measure $e^{-u}\,\mathrm{d}u$ (magnitude)



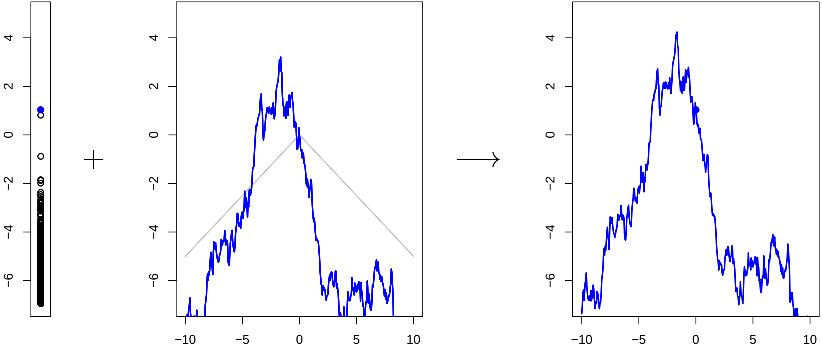
Point Process Construction (Brown & Resnick 1977)

- $\{U_k,\, k \in \mathcal{N}\}$: PPP on $\mathbb{R}$ with intensity measure $e^{-u}\,\mathrm{d}u$ (magnitude)
- $W(\cdot)$: standard Brownian motion (BM)
  $(W_k(\cdot) - |\cdot|/2) \sim_{i.i.d} (W(\cdot) - |\cdot|/2)$ (spatial course)



Point Process Construction (Brown & Resnick 1977)

$$X(t) = \max_{k \in \mathcal{N}} \left(U_k + W_k(t) - \frac{|t|}{2} \right), \quad t \in \mathbb{R}$$



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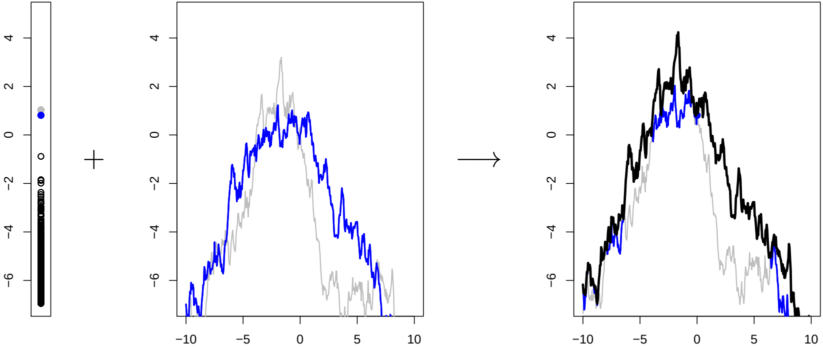

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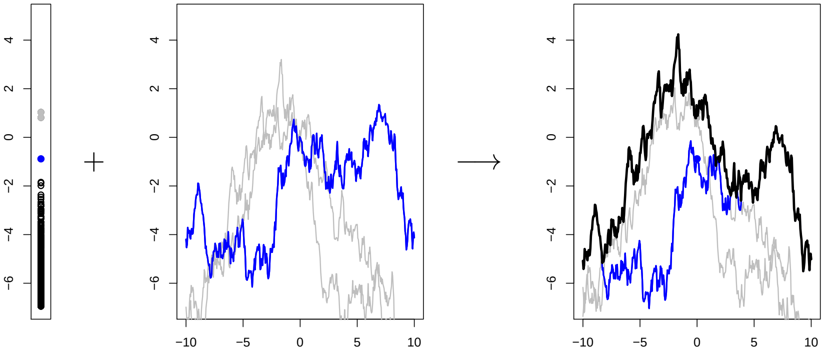

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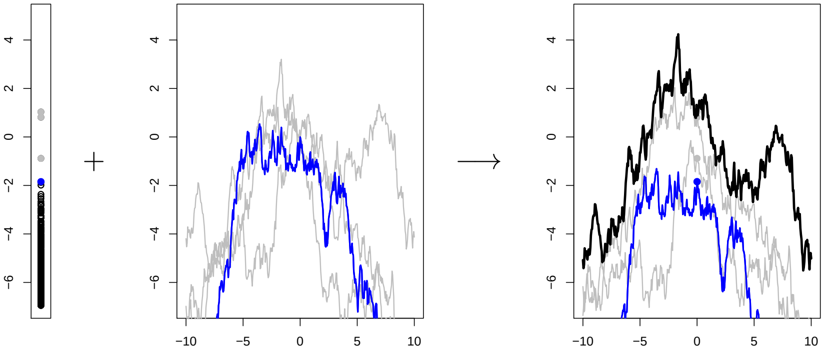
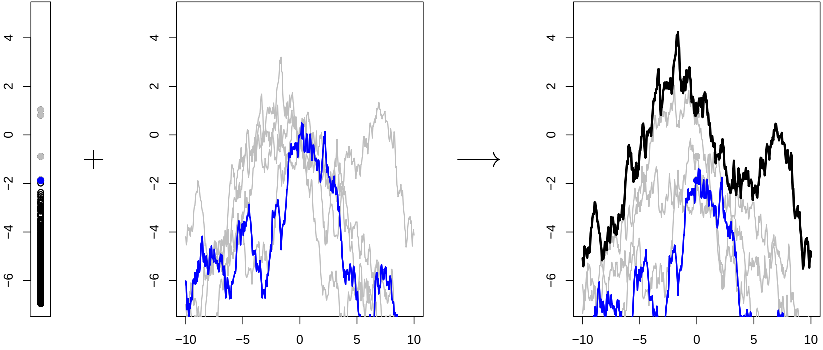
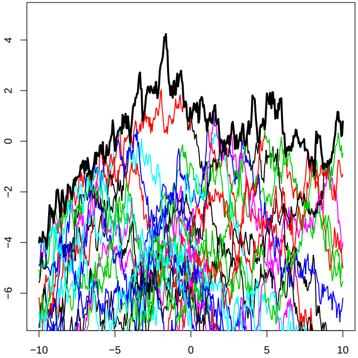


Point Process Construction (Brown & Resnick 1977)

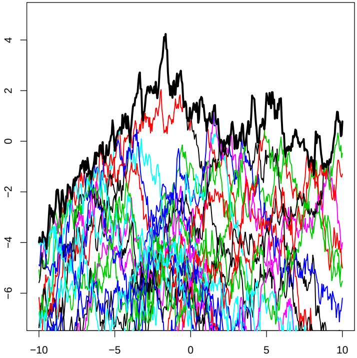
$$X(t) = \max_{k \in \mathcal{N}} \left(U_k + W_k(t) - \frac{|t|}{2} \right), \quad t \in \mathbb{R}$$





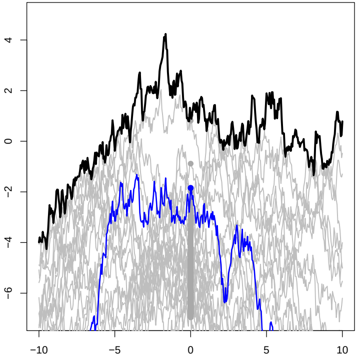
$$X(t) = \max_{k \in \mathcal{N}} \left(U_k + W_k(t) - \frac{|t|}{2} \right)$$

X is max-stable



$$X(t) = \max_{k \in \mathcal{N}} \left(U_k + W_k(t) - \frac{|t|}{2} \right)$$

X is max-stable and stationary



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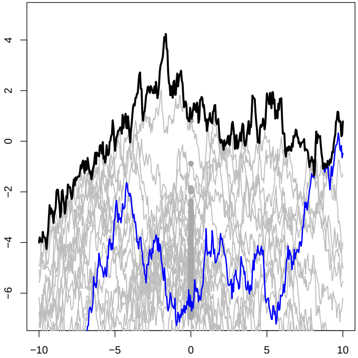

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X is max-stable and stationary

Generalization

(cf. Kabluchko *et al.* 2009)

- $\{U_k\}_{k\in\mathcal{N}}$: Poisson point process with intensity $e^{-u}\,\mathrm{d}u$
- $W(\cdot)$: centered Gaussian process on $\mathbb{R}^d$ s.t.

  variogram

  $\gamma(s,t) = \mathrm{Var}(W(s) - W(t))$

  depends on $s - t \in \mathbb{R}^d$ only
- $\{W_k(\cdot)\}_{k\in\mathcal{N}}$: independent copies of W

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$$X(t) = \max_{k\in\mathcal{N}} \left(U_k + W_k(t) - \mathrm{Var}(W_k(t))/2\right), \quad t \in \mathbb{R}^d,$$

X is called Brown-Resnick process associated to the variogram γ.

Multivariate Generalization

(cf. Stucki & Molchanov, 2013, Oesting et al., 2013)

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variogram

$\gamma(s,t) = (\mathrm{Var}(W\quad(s) - W\quad(t)))$

depends on $s - t \in \mathbb{R}^d$ only

- $\{W_k(\cdot)\}_{k\in\mathcal{N}}$: independent copies of W

$$X\quad(t) = \max_{k\in\mathcal{N}} \left(U_k + W_k\quad(t) - \mathrm{Var}(W_k\quad(t))/2\right), \quad t \in \mathbb{R}^d,$$

Then,
- X is max-stable and stationary
- law of X depends on γ only

Multivariate Generalization

(cf. Stucki & Molchanov, 2013, Oesting et al., 2013)

- $\{U_k\}_{k\in\mathcal{N}}$: Poisson point process with intensity $e^{-u}\,\mathrm{d}u$
- $W(\cdot) = (W^{(1)}(\cdot), W^{(2)}(\cdot))$: centered Gaussian process s.t.

pseudo-variogram

$$\gamma(s,t) = (\mathrm{Var}(W^{(i)}(s) - W^{(j)}(t)))_{i,j=1,2}$$

depends on $s - t \in \mathbb{R}^d$ only

- $\{W_k(\cdot)\}_{k\in\mathcal{N}}$: independent copies of W

$$X^{(i)}(t) = \max_{k\in\mathcal{N}} \left(U_k + W_k^{(i)}(t) - \mathrm{Var}(W_k^{(i)}(t))/2 \right), \quad t \in \mathbb{R}^d,\ i = 1,2,$$

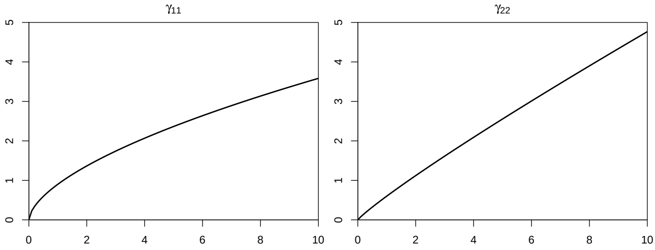
Then,
- X is max-stable and stationary (as bivariate process)
- law of X depends on γ only

What does a pseudo-variogram look like?

$$\gamma(s,t) = (\mathrm{Var}(W^{(i)}(s) - W^{(j)}(t)))_{1 \le i,j \le 2}$$

Question: Can a pseudo-variogram have the form

$$\gamma(t+h,t) = \begin{pmatrix} \|h\|^{\alpha} & ? \\ ? & \|h\|^{\beta} \end{pmatrix}, \quad 0 < \alpha \neq \beta \le 2?$$



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$$\gamma(s,t) = (\mathrm{Var}(W^{(i)}(s) - W^{(j)}(t)))_{1\le i,j\le 2}$$

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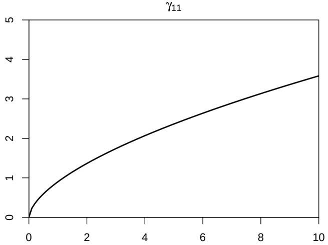


Answer: No!

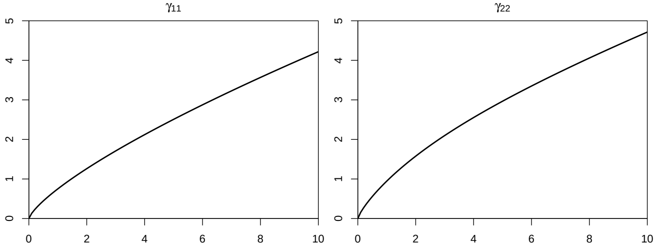
What does a pseudo-variogram look like? (cont'd)

Theorem (Oesting et al.,2013)

Let $\gamma(s,t)$ a pseudo-variogram that depends on $s-t$ only.
Then, γ is of the form

$$\sqrt{\gamma(t+h,t)} = \begin{pmatrix} \sqrt{\gamma^*(h)} & \sqrt{\gamma^*(h)} \\ \sqrt{\gamma^*(h)} & \sqrt{\gamma^*(h)} \end{pmatrix} + \begin{pmatrix} f_{11}(h) & f_{12}(h) \\ f_{21}(h) & f_{22}(h) \end{pmatrix}, \quad t,h \in \mathbb{R}^d,$$

for some univariate variogram γ^* and bounded functions $(f_{ij}(\cdot))_{1\le i,j\le 2}$.



Construction Principle:

- $Y(\cdot)$: univariate Gaussian process with stationary increments and variogram γ^*
- $V(\cdot) = (V^{(1)}(\cdot), V^{(2)}(\cdot))$: bivariate stationary Gaussian process with covariance function

$$C(h) = \begin{pmatrix} C_{11}(h) & C_{12}(h) \\ C_{21}(h) & C_{22}(h) \end{pmatrix}$$

$W(\cdot) = (Y(\cdot) + V^{(1)}(\cdot), Y(\cdot) + V^{(2)}(\cdot))$ has a pseudo-variogram

$$\gamma(h) = (\gamma^*(h) + C_{ii}(0) + C_{jj}(0) - 2C_{ij}(h))_{i,j=1,2}\,.$$

Full model

- **Reminder:**
  marginals of $V_{\mathsf{max}}^{\mathrm{obs}}$ and $V_{\mathsf{max}}^{\mathrm{pred}}$ are modelled by GEVs (parameters estimated MLE)

$V_{\mathsf{max}}^{\mathrm{obs}}$ (GEV)	$V_{\mathsf{max}}^{\mathrm{pred}}$ (GEV)

Full model

- **Reminder:**
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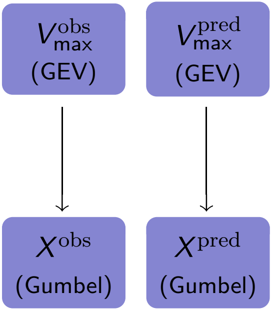

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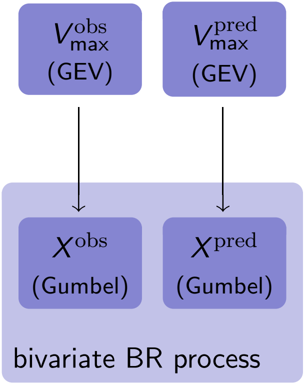
Full model

- **Reminder:**
  marginals of $V_{\max}^{\text{obs}}$ and $V_{\max}^{\text{pred}}$ are modelled by GEVs (parameters estimated MLE)
- data are transformed to standard Gumbel margins ($\rightsquigarrow X^{\text{obs}}, X^{\text{pred}}$)
- standardized observation and forecast are jointly modelled by bivariate BR process
  - dependence in space
  - dependence between observations and forecast

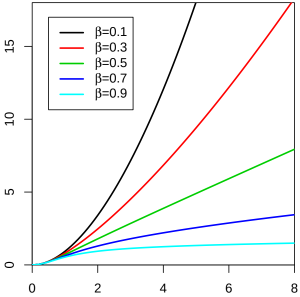


Bivariate Variogram Model

$$\gamma(h) = \left(\gamma^*(h) + C_{ii}(0) + C_{jj}(0) - 2C_{ij}(h)\right)_{i,j=1,2}.$$

- γ^*: variogramm of power law type

$$\gamma^*(h) = \frac{\|h\|^2}{(1+\|h\|^2)^\beta}, \quad \beta \in (0,1)$$



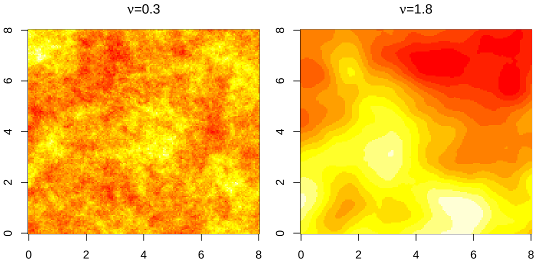
Bivariate Variogram Model

$$\gamma(h) = \left(\gamma^*(h) + C_{ii}(0) + C_{jj}(0) - 2C_{ij}(h)\right)_{i,j=1,2}.$$

- C: bivariate Matérn model (Gneiting et. al., 2010)

$$C_{ij}(h) = \rho_{ij}\sigma_i\sigma_j \frac{2^{1-\nu_{ij}}}{\Gamma(\nu_{ij})}(a\|h\|)^{\nu_{ij}} K_{\nu_{ij}}(a\|h\|)$$

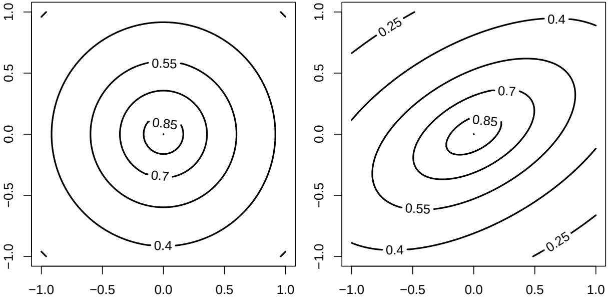
for $\sigma_1, \sigma_2 \geq 0$, $a, \nu_{11}, \nu_{22} > 0$, $\nu_{12} = (\nu_{11} + \nu_{22})/2$ and suitable ρ_{ij}



Bivariate Variogram Model

$$\gamma(h) = \left(\gamma^*(h) + C_{ii}(0) + C_{jj}(0) - 2C_{ij}(h)\right)_{i,j=1,2}.$$

- γ^*: $\quad \gamma^*(h) = \|Ah\|^2 \Big/ (1 + \|Ah\|^2)^\beta, \quad \beta \in (0,1)$
- C: $\quad C_{ij}(h) = \rho_{ij}\sigma_i\sigma_j 2^{1-\nu_{ij}}\Gamma(\nu_{ij})^{-1}\Big(a\|Ah\|\Big)^{\nu_{ij}}\mathcal{K}_{\nu_{ij}}\Big(a\|Ah\|\Big)$
- introduce anisotropy matrix A (dilation/rotation)



Estimation of Dependence Structure

Extremal coefficient function for bivariate processes:

$$\mathbb{P}(X^{\mathrm{obs}}(s) \le x, X^{\mathrm{obs}}(t) \le x) = \mathbb{P}(X^{\mathrm{obs}}(0) \le x)^{\theta^{\mathrm{obs,obs}}(s,t)}$$
$$\mathbb{P}(X^{\mathrm{obs}}(s) \le x, X^{\mathrm{pred}}(t) \le x) = \mathbb{P}(X^{\mathrm{pred}}(0) \le x)^{\theta^{\mathrm{obs,pred}}(s,t)}$$
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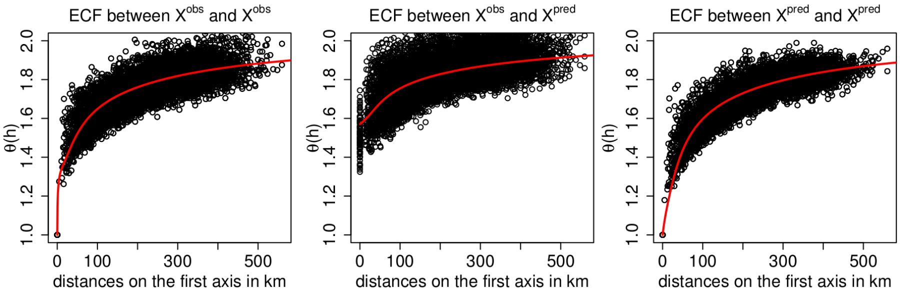
$$\mathbb{P}(X^{\mathrm{pred}}(s) \le x, X^{\mathrm{pred}}(t) \le x) = \mathbb{P}(X^{\mathrm{pred}}(0) \le x)^{\theta^{\mathrm{pred,pred}}(s,t)}$$

Estimation: components are estimated separately via F-madogram

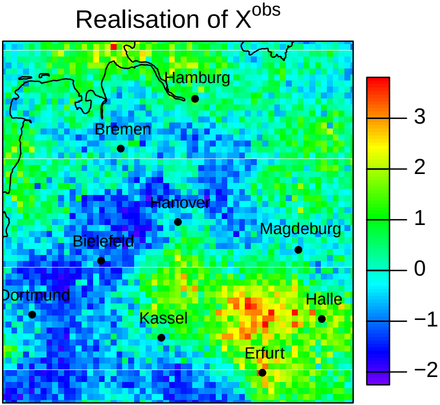
ECF for bivariate BR processes:

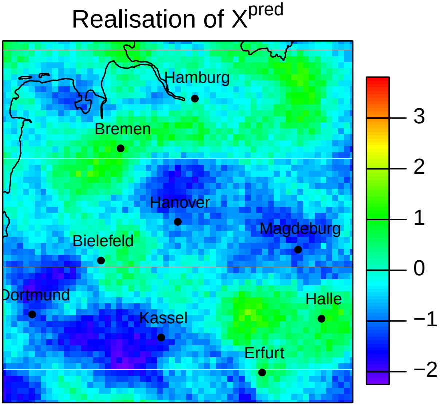
$$\begin{pmatrix} \theta^{\mathrm{obs,obs}}(s,t) & \theta^{\mathrm{obs,pred}}(s,t) \\ \theta^{\mathrm{pred,obs}}(s,t) & \theta^{\mathrm{pred,pred}}(s,t) \end{pmatrix} = \left(2\Phi\left(\frac{\sqrt{\gamma_{ij}(s-t)}}{2}\right)\right)_{i,j=1,2}$$

Extremal Coefficient Function



Unconditional simulation of the Brown-Resnick process:





Outlook: Post-Processing of the Forecast

Given Data

- historical observation and forecast data
- forecast for today

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Bivariate BR process provides a model for observed wind gusts conditional on forecast
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Bivariate BR process provides a model for observed wind gusts conditional on forecast
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1. standardized forecast $V_{\mathrm{max}}^{\mathrm{pred}}$ to standard Gumbel margins $\rightsquigarrow$ X^{pred}
2. simulate realizations of $X^{\mathrm{pred}} \mid X^{\mathrm{obs}}$
3. transform X^{obs} from Gumbel to GEV margins $\rightsquigarrow$ $V_{\mathrm{max}}^{\mathrm{obs}}$