Modelling multivariate counts varying continuously in space

Alexandra M. Schmidt

IM - UFRJ, Brazil

Homepage: `www.dme.ufrj.br/~alex`

joint work with Marco A. Rodríguez
UQTR, Canada

in Bayesian Statistics 9, pp. 611-628

PASI 2014, Workshop on Multivariate Spatial Stats

Búzios, Brazil, June 2014

Outline

Motivation and aims

Distr. multiv. counts

Proposed model

Inference procedure

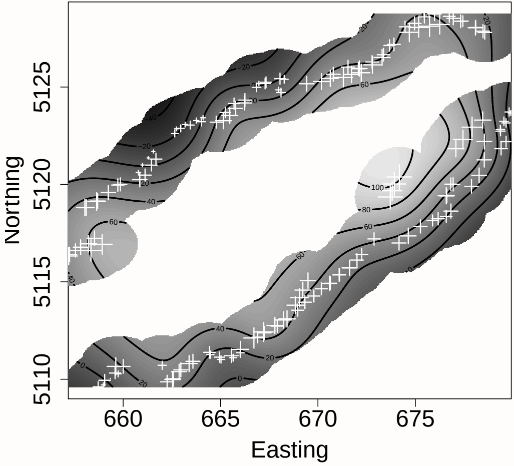
Results
- Model comparison
- Results

Conclusions

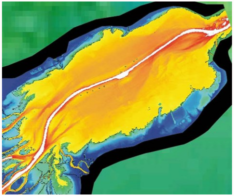
Main references

Outline of the talk

- Motivation and aims
- Distributions for multivariate counts: a brief review
- Our proposed model
- Covariates in covariance structure
- Model comparison and results
- Discussion



Sampling locations in Lake St. Pierre (+) and geodetic depth contours (curves)



Central navigation channel (deep; strong current)

- On each sampling date, measurements were made at a cluster of locations on a shore, selected randomly among all clusters on that shore
- Sampling dates were unevenly spaced in time over a period of 70 days; the North and South shores were visited in alternation on consecutive sampling dates

Outline

Motivation and aims

Distr. multiv. counts

Proposed model

Inference procedure

Results
Model comparison
Results

Conclusions

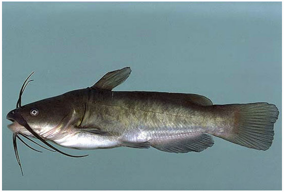
Main references

Fish were collected by electrofishing: fish are attracted to anodes hanging from the booms

Outline
Motivation and aims
Distr. multiv. counts
Proposed model
Inference procedure
Results
Model comparison
Results
Conclusions
Main references

Most abundant fish species observed in Lake St. Pierre

(a) Yellow perch



(b) Brown bullhead

(c) Golden shiner

(d) Pumpkinseed

Outline

Motivation and aims

Distr. multiv. counts

Proposed model

Inference procedure

Results
- Model comparison
- Results

Conclusions

Main references

Main aims

- Assess the influence of local habitat (characterized by environmental covariates water depth, water transparency, substrate, vegetation) on the abundance of fish species
- Determine whether species abundances are correlated across space and among themselves
- Understand the spatial distribution of each species

Outline
Motivation and aims
Distr. multiv. counts
Proposed model
Inference procedure
Results
Model comparison
Results
Conclusions
Main references

Distributions for multivariate counts

Probability distributions for multivariate counts can be defined through:

the sum of independent Poisson distributions $\rightarrow$ does not account for overdispersion or negative covariance;

Outline

Motivation and aims

Distr. multiv. counts

Proposed model

Inference procedure

Results
Model comparison
Results

Conclusions

Main references

Distributions for multivariate counts

Outline

Motivation and aims

Distr. multiv. counts

Proposed model

Inference procedure

Results

Model comparison

Results

Conclusions

Main references

Probability distributions for multivariate counts can be defined through:

continuous mixtures of independent Poisson distributions. Let $\mathbf{Y} = (Y_1, \cdots, Y_K)$, with $Y_k \mid \delta_k \sim Poi(\delta_k)$, $k = 1, \cdots, K$, conditionally independent, then

$$f(y \mid \theta) = \int \prod_{k=1}^{K} p(y_k \mid \delta_k) g(\delta \mid \theta) d\delta,$$

where $\delta = (\delta_1, \cdots, \delta_K)$.

- $g(.)$ can be the pdf of a multivariate gamma $\rightarrow$ accounts for overdispersion but not for negative covariance
- $g(.)$ can be the pdf of a multivariate log-normal distribution

Multivariate Poisson as a Sum of Independent Poisson Random Variables

Let

$$
\begin{aligned}
Y_1 &= W_1 + W_{12} + W_{13} + W_{123} \\
Y_2 &= W_2 + W_{12} + W_{23} + W_{123} \\
Y_3 &= W_3 + W_{23} + W_{13} + W_{123},
\end{aligned}
$$

with $W_i \sim Poi(\lambda_i)$, $W_{ij} \sim Poi(\lambda_{ij})$ $W_{ijl} \sim Poi(\lambda_{ijl})$, $i, j, l = \{1, 2, 3\}$, $i < j < l$.

More generally, $\mathbf{Y} = \mathbf{BW}$, such that, $E(\mathbf{Y}) = \mathbf{B}\boldsymbol{\lambda}$ and $V(\mathbf{Y}) = \mathbf{B}\boldsymbol{\Sigma}\mathbf{B}^T$, where $\boldsymbol{\Sigma} =$ diag $(\lambda_1, \cdots, \lambda_q)$.

- Mean and variance of Y_i are assumed identical.
- Model only accounts for positive covariance structures.

Outline

Motivation and aims

Distr. multiv. counts

Proposed model

Inference procedure

Results

Model comparison

Results

Conclusions

Main references

Multivariate Count Distributions Based on Mixtures

Let $\mathbf{Y} = (Y_1, \cdots, Y_K)$, with $Y_k \mid \delta_k \sim Poi(\delta_k)$, $k = 1, \cdots, K$, conditionally independent, then

$$f(y \mid \theta) = \int \prod_{k=1}^{K} p(y_k \mid \delta_k) g(\delta \mid \theta) d\delta,$$

where $\delta = (\delta_1, \cdots, \delta_K)$.

Outline

Motivation and aims

Distr. multiv. counts

Proposed model

Inference procedure

Results

Model comparison

Results

Conclusions

Main references

Multivariate Poisson-Gamma Mixture

A multivariate gamma distribution for a K-dimensional random vector δ can be obtained by defining

$$\boldsymbol{\delta} = \mathbf{B}\,\mathbf{W} = \begin{pmatrix} \frac{b_0}{b_1} & 1 & 0 & \cdots & 0 \\ \frac{b_0}{b_2} & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{b_0}{b_K} & 0 & 0 & \cdots & 1 \end{pmatrix} \begin{pmatrix} W_0 \\ W_1 \\ \vdots \\ W_K \end{pmatrix}, \qquad (1)$$

with $W_k \sim Ga(a_k, b_k)$ for $k = 0, 1, 2, \cdots, K$; W_k independent among themselves (Mathai & Moschopoulos, 1991).
It is easy to show that Cov$(\delta_k, \delta_l) = \frac{a_0}{b_k\, b_l}$, $k, l = 1, 2, \cdots, K$.

Outline
Motivation and aims
Distr. multiv. counts
Proposed model
Inference procedure
Results
Model comparison
Results
Conclusions
Main references

If $\mathbf{Y} = (Y_1, \cdots, Y_K)$, such that $Y_i | \delta_i \sim Poi(\delta_i)$, independently, and $\boldsymbol{\delta} = \mathbf{BW}$ it can be shown that, marginally,

$$E(Y_i) = \frac{a_i}{b_i} + \frac{a_0}{b_i} = \mu_i \qquad i, j = 1, \cdots, K$$
$$V(Y_i) = \mu_i + \frac{a_0}{b_i^2} + \frac{a_i}{b_i}$$
$$\text{Cov}(Y_i, Y_j) = \frac{a_0}{b_i \, b_j}.$$

- It accounts for overdispersion,
- however, it only captures positive covariance structures.

Outline
Motivation and aims
Distr. multiv. counts
Proposed model
Inference procedure
Results
Model comparison
Results
Conclusions
Main references

Multivariate Poisson-lognormal mixture (Aitchison & Ho 1989)

Assume that

$$\log \boldsymbol{\delta} \sim N_K(\boldsymbol{\mu}, \boldsymbol{\Sigma}),$$

where $\boldsymbol{\Sigma}$ has elements σ_{kl}. Assuming $Y_k \mid \delta_k \sim Poi(\delta_k)$ independently.
Marginally,

$$\begin{aligned} E(Y_k) &= \exp\left(\mu_k + \frac{1}{2}\sigma_{kk}\right) = \alpha_k \\ V(Y_k) &= \alpha_k + \alpha_k^2\{\exp(\sigma_{kk}) - 1\} \\ \text{Cov}(Y_k, Y_l) &= \alpha_k\alpha_l\{\exp(\sigma_{kl}) - 1\}, \; k, l = 1, \cdots, K. \end{aligned}$$

- This model accounts for overdispersion
- Allows for both positive and negative covariance structures

Outline
Motivation and aims
Distr. multiv. counts
Proposed model
Inference procedure
Results
Model comparison
Results
Conclusions
Main references

Multivariate Poisson-lognormal mixture (Aitchison & Ho 1989)

Assume that

$$\log \boldsymbol{\delta} \sim N_K(\boldsymbol{\mu}, \boldsymbol{\Sigma}),$$

where $\boldsymbol{\Sigma}$ has elements σ_{kl}. Assuming $Y_k \mid \delta_k \sim Poi(\delta_k)$ independently.
This approach is appealing, as we can write

$$\begin{aligned} \log \delta_k &= \beta X_k + \epsilon_k, \; k = 1, \cdots, K \\ \epsilon &\sim N_K(0, \Sigma) \end{aligned}$$

- Chib and Winkelmann (2001) were the first to provide a full Bayesian treatment of this model
- A similar approach has been widely used for multiple disease mapping (e.g., Carlin and Banerjee 2003, Gelfand and Vounatsou 2003, Jin et al. 2007; good overview in Lawson 2009)

Outline
Motivation and aims
Distr. multiv. counts
Proposed model
Inference procedure
Results
Model comparison
Results
Conclusions
Main references

Proposed model

Let $Y_k(s_{t_j})$ represent the number of individuals (counts) of species k, $k = 1, \cdots, K$, observed at location s_{t_j}, $j = 1, \cdots, n_t$ and time $t = 1, \cdots, T$. We assume

$$Y_k(s_{t_j}) \mid \theta_k(s_{t_j}), \delta_k(s_{t_j}) \sim Poi(\theta_k(s_{t_j})\delta_k(s_{t_j})),$$

where

$$\log \theta_k(s_{t_j}) = \mathbf{X}_k(s_{t_j})\boldsymbol{\beta}_k$$

which captures local habitat structures.

The parameter vector $\boldsymbol{\delta}(s_{t_j}) = (\delta_1(s_{t_j}), \cdots, \delta_K(s_{t_j}))'$ plays the role of the mixing component.

Outline
Motivation and aims
Distr. multiv. counts
Proposed model
Inference procedure
Results
Model comparison
Results
Conclusions
Main references

Proposed model - Mixing component

$$\log \delta_k(s_{t_j}) = \gamma_k(s_t) + \nu_k(s_{t_j}),$$

log(mixing component) = temporal effect + local effect

Outline
Motivation and aims
Distr. multiv. counts
Proposed model
Inference procedure
Results
Model comparison
Results
Conclusions
Main references

Proposed model - Mixing component

$$\log \delta_k(s_{t_j}) = \gamma_k(s_t) + \nu_k(s_{t_j}),$$

Modelling the temporal effect:

Let $\gamma(s_t) = (\gamma_1(s_t) \cdots \gamma_K(s_t))'$.
We assume

$$\gamma(s_t) \sim N_K(0, \Omega), \quad \forall\, t = 1, \cdots, T,$$

where Ω captures the covariance among species at each time t.

Proposed model - Mixing component

Outline
Motivation and aims
Distr. multiv. counts
Proposed model
Inference procedure
Results
Model comparison
Results
Conclusions
Main references

$$\log \delta_k(s_{t_j}) = \gamma_k(s_t) + \nu_k(s_{t_j}),$$

Modelling the local effect:

Let $\boldsymbol{\nu}(s_{t_j}) = (\nu_1(s_{t_j}), \cdots, \nu_K(s_{t_j}))$, following the LMC (Gelfand et al 2004)

$$\boldsymbol{\nu}(s_{t_j}) = \mathbf{A}\boldsymbol{\omega}(s_{t_j}).$$

$\mathbf{A}$ is lower triangular and $\boldsymbol{\omega}(s_{t_j}) = (\omega_1(s_{t_j}) \cdots \omega_K(s_{t_j}))'$ such that, independently,

$$\omega_k(.) \sim GP(0, \rho(\boldsymbol{\vartheta}_k; d)), \quad k = 1, \cdots, K.$$

Then

$$Cov(\boldsymbol{\nu}(s), \boldsymbol{\nu}(s')) = \sum_{k=1}^{K} \rho(\boldsymbol{\vartheta}_k; d)\mathbf{M}_k, \quad \text{with } \mathbf{M}_k = \mathbf{a}_k\mathbf{a}_k^T$$

Joint distribution of $\log \boldsymbol{\delta}$

Let $\boldsymbol{\delta}$ be the nK-dimensional vector containing the elements of the mixing component, $\boldsymbol{\delta} = (\delta(s_{1_1}), \cdots, \delta(s_{T_{n_T}}))'$.

- Separable model ($\rho(\vartheta; d)$, for $k = 1, \cdots, K$)

$$\log \boldsymbol{\delta} \mid \boldsymbol{\gamma} \sim N\left((I_K \otimes C)\boldsymbol{\gamma}\, ,\, \mathbf{R} \otimes \mathbf{M} \right),$$

  where $\mathbf{M} = \mathbf{A}\mathbf{A}^T$.

- Non-separable model ($\rho(\vartheta_k; d)$)

$$\log \boldsymbol{\delta} \mid \boldsymbol{\gamma} \sim N\left((I_K \otimes C)\boldsymbol{\gamma}\, ,\, \sum_{j=1}^{K} \mathbf{R}_j \otimes \mathbf{M}_k \right).$$

We now discuss the specification of the spatial correlation function (elements of $\mathbf{R}$).

Outline
Motivation and aims
Distr. multiv. counts
Proposed model
Inference procedure
Results
Model comparison
Results
Conclusions
Main references

Joint distribution of $\log \boldsymbol{\delta}$

Let $\boldsymbol{\delta}$ be the nK-dimensional vector containing the elements of the mixing component, $\boldsymbol{\delta} = (\delta(s_{1_1}), \cdots, \delta(s_{T_{n_T}}))'$.

- Separable model ($\rho(\boldsymbol{\vartheta}; d)$, for $k = 1, \cdots, K$)

$$\log \boldsymbol{\delta} \mid \boldsymbol{\gamma} \sim N\left((I_K \otimes C)\boldsymbol{\gamma}\,,\, \mathbf{R} \otimes \mathbf{M} \right),$$

  where $\mathbf{M} = \mathbf{A}\mathbf{A}^T$.

- Non-separable model ($\rho(\boldsymbol{\vartheta}_k; d)$)

$$\log \boldsymbol{\delta} \mid \boldsymbol{\gamma} \sim N\left((I_K \otimes C)\boldsymbol{\gamma}\,,\, \sum_{j=1}^{K} \mathbf{R}_j \otimes \mathbf{M}_k \right).$$

We now discuss the specification of the spatial correlation function (elements of $\mathbf{R}$).

Outline
Motivation and aims
Distr. multiv. counts
Proposed model
Inference procedure
Results
Model comparison
Results
Conclusions
Main references

Joint distribution of $\log \boldsymbol{\delta}$

Let $\boldsymbol{\delta}$ be the nK-dimensional vector containing the elements of the mixing component, $\boldsymbol{\delta} = (\delta(s_{1_1}), \cdots, \delta(s_{T_{n_T}}))'$.

- Separable model ($\rho(\boldsymbol{\vartheta}; d)$, for $k = 1, \cdots, K$)

$$\log \boldsymbol{\delta} \mid \boldsymbol{\gamma} \sim N\left((I_K \otimes C)\boldsymbol{\gamma}\,,\, \mathbf{R} \otimes \mathbf{M} \right),$$

  where $\mathbf{M} = \mathbf{A}\mathbf{A}^T$.

- Non-separable model ($\rho(\boldsymbol{\vartheta}_k; d)$)

$$\log \boldsymbol{\delta} \mid \boldsymbol{\gamma} \sim N\left((I_K \otimes C)\boldsymbol{\gamma}\,,\, \sum_{j=1}^{K} \mathbf{R}_j \otimes \mathbf{M}_k \right).$$

We now discuss the specification of the spatial correlation function (elements of $\mathbf{R}$).

Outline

Motivation and aims

Distr. multiv. counts

Proposed model

Inference procedure

Results

Model comparison

Results

Conclusions

Main references

Isotropic model

The correlation structure of $\omega_k(.)$ is given by

$$\rho(s, s'; \boldsymbol{\vartheta}_k) = \exp(-\phi_k ||s - s'||).$$

with parameter $\vartheta_k = \phi_k$.

Alex Schmidt
IM-UFRJ
Brazil

PASI 2014
Multivariate Spatial Stats
Búzios, Brazil, 2014

Outline
Motivation and aims
Distr. multiv. counts
Proposed model
Inference procedure
Results
Model comparison
Results
Conclusions
Main references

Elliptical anisotropy

The correlation structure of $\omega_k(.)$ is given by

$$\rho(s, s'; \boldsymbol{\vartheta}_k) = \exp(-\phi_k ||d_k(s) - d_k(s')||),$$

with $d_k(s) = s\, D_k$, and

$$D_k = \begin{bmatrix} \cos\psi_{A_k} & -\sin\psi_{A_k} \\ \sin\psi_{A_k} & \cos\psi_{A_k} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & \psi_{R_k}^{-1} \end{bmatrix},$$

where ψ_{A_k} is the *anisotropy angle* and $\psi_{R_k} > 1$ is the *anisotropy ratio*, with parameters $\boldsymbol{\vartheta}_k = (\phi_k, \psi_{A_k}, \psi_{R_k})$.

Outline
Motivation and aims
Distr. multiv. counts
Proposed model
Inference procedure
Results
Model comparison
Results
Conclusions
Main references

Covariates in the covariance structure (Schmidt et al. 2011)

Now location has a broader interpretation.
Let $\mathbf{w} = (long, lat, z_3, \cdots, z_C)$, and

$$\rho(\mathbf{w}, \mathbf{w}'; \boldsymbol{\vartheta}_k) = \exp\left(-\sqrt{(\mathbf{w} - \mathbf{w}')^T \, \Phi^{-1} \, (\mathbf{w} - \mathbf{w}')}\right).$$

Similar to assuming a GP in R^C, generating an anisotropic correlation function in R^2 ("geographical" space).

Our model assumes

$$\rho(\mathbf{w}, \mathbf{w}'; \vartheta_k) = \exp\left(-\phi_1 ||s - s'|| - \phi_2 |z - z'|\right),$$

where z is *geodetic depth* (because lake level fluctuates, local habitat travels, but geodetic depth is fixed).

Outline
Motivation and aims
Distr. multiv. counts
Proposed model
Inference procedure
Results
Model comparison
Results
Conclusions
Main references

Covariates in the covariance structure (Schmidt et al. 2011)

Now location has a broader interpretation.
Let $\mathbf{w} = (long, lat, z_3, \cdots, z_C)$, and

$$\rho(\mathbf{w}, \mathbf{w}'; \boldsymbol{\vartheta}_k) = \exp\left(-\sqrt{(\mathbf{w} - \mathbf{w}')^T \, \Phi^{-1} \, (\mathbf{w} - \mathbf{w}')}\right).$$

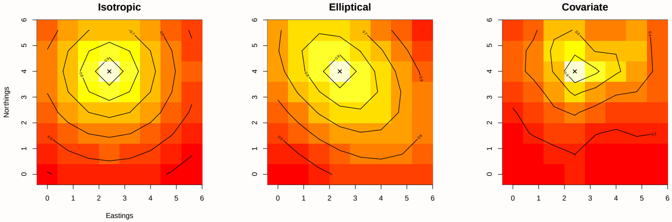
Similar to assuming a GP in R^C, generating an anisotropic correlation function in R^2 ("geographical" space).

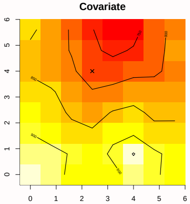
Our model assumes

$$\rho(\mathbf{w}, \mathbf{w}'; \boldsymbol{\vartheta}_k) = \exp\left(-\phi_1 ||s - s'|| - \phi_2 |z - z'|\right),$$

where z is *geodetic depth* (because lake level fluctuates, local habitat travels, but geodetic depth is fixed).

Comparing correlation structures





Likelihood function

Let
$\mathbf{y} = (\mathbf{y}(s_{1_1}), \mathbf{y}(s_{1_2}), \cdots, \mathbf{y}(s_{1_{n_1}}), \cdots, \mathbf{y}(s_{T_1}), \cdots, \mathbf{y}(s_{T_{n_T}}))'$
be the observed counts over the sampling period at each location s_{t_j}, $t = 1, \cdots, T$, $j = 1, \cdots, n_t$.

The likelihood function is

$$l(\mathbf{y} \mid \boldsymbol{\theta}, \boldsymbol{\delta}) \propto \prod_{t=1}^{T} \prod_{j=1}^{n_t} \prod_{k=1}^{K} \exp\left\{-\theta_k(s_{t_j})\, \delta_k(s_{t_j})\right\} \left[\theta_k(s_{t_j})\, \delta_k(s_{t_j})\right]^{y_k(s_{t_j})} .$$

Outline

Motivation and aims

Distr. multiv. counts

Proposed model

Inference procedure

Results
Model comparison
Results

Conclusions

Main references

Prior specification

- $\boldsymbol{\beta} \sim N(0, \sigma^2_\beta I_p)$
- $\phi_{ij} \sim IG(2, b)$, $i = 1, 2$, $j = 1, \cdots, K$, and b is fixed such that the practical range (correlation = 0.05) is reached at half of the maximum distance between observations
- $\boldsymbol{\Omega} \sim IW(K + 1, c\, I_k)$
- $\psi_R \sim Pareto(1, 2)$, $\psi_A \sim U(0, \pi)$
- $a_{ij} \sim N(0, 5)$ and $\log a_{ii} \sim N(0, 5)$

Outline
Motivation and aims
Distr. multiv. counts
Proposed model
Inference procedure
Results
Model comparison
Results
Conclusions
Main references

MCMC sampling scheme

We reparametrize the model such that $\varphi_k(s_{t_j}) = \theta_k(s_{t_j})\,\delta(s_{t_j})$, and

$$\log \varphi_k(s_{t_j}) = X_k^*(s_{t_j})\boldsymbol{\beta}_k^* + W_k(s_{t_j})$$

and $W_k(s_{t_j}) = \beta_{1k} + \gamma_k(s_t) + \nu_k(s_{t_j})$, where $X_k^*(.)$ does not have a column of ones, and $\boldsymbol{\beta}_k^* = (\beta_{2k}, \cdots, \beta_{p_k k})^T$.

- $\boldsymbol{\beta}_k$ and $W_k(.)$ M-H algorithm proposed by Gamerman (1997)
- $\boldsymbol{\beta}_{1.} = (\beta_{11}, \cdots, \beta_{1K})'$ and γ normal posterior full conditionals
- $\boldsymbol{\Omega}$ follows an inverse-Wishart posterior full conditional
- ϕ_1, ϕ_2, ψ_R, ψ_A, and elements of $\mathbf{A}$ are sampled through M-H steps

Outline
Motivation and aims
Distr. multiv. counts
Proposed model
Inference procedure
Results
Model comparison
Results
Conclusions
Main references

Fitted models

M1: Separable isotropic covariance structure

M2: Separable elliptical anisotropy covariance structure

M3: Separable covariate-dependent (z = geodetic depth) covariance structure

M4: Non-separable isotropic covariance structure

M5: Non-separable elliptical anisotropy covariance structure

M6: Non-separable, covariate-dependent (z = geodetic depth) covariance structure

- $\gamma(s_t)$ assumed to be independent across time and common to both shores
- We allow for different local random effects for the North and South shores (Glémet and Rodríguez 2007; Schmidt et al. 2010b)

Outline
Motivation and aims
Distr. multiv. counts
Proposed model
Inference procedure
Results
Model comparison
Results
Conclusions
Main references

Fitted models

M1: Separable isotropic covariance structure

M2: Separable elliptical anisotropy covariance structure

M3: Separable covariate-dependent ($z =$ geodetic depth) covariance structure

M4: Non-separable isotropic covariance structure

M5: Non-separable elliptical anisotropy covariance structure

M6: Non-separable, covariate-dependent ($z =$ geodetic depth) covariance structure

- $\gamma(s_t)$ assumed to be independent across time and common to both shores
- We allow for different local random effects for the North and South shores (Glémet and Rodríguez 2007; Schmidt et al. 2010b)

Alex Schmidt
IM-UFRJ
Brazil

Outline
Motivation and aims
Distr. multiv. counts
Proposed model
Inference procedure
Results
Model comparison
Results
Conclusions
Main references

Fitted models

M1: Separable isotropic covariance structure

M2: Separable elliptical anisotropy covariance structure

M3: Separable covariate-dependent ($z =$ geodetic depth) covariance structure

M4: Non-separable isotropic covariance structure

M5: Non-separable elliptical anisotropy covariance structure

M6: Non-separable, covariate-dependent ($z =$ geodetic depth) covariance structure

- $\gamma(s_t)$ assumed to be independent across time and common to both shores
- We allow for different local random effects for the North and South shores (Glémet and Rodríguez 2007; Schmidt et al. 2010b)

Outline
Motivation and aims
Distr. multiv. counts
Proposed model
Inference procedure
Results
Model comparison
Results
Conclusions
Main references

Fitted models

M1: Separable isotropic covariance structure

M2: Separable elliptical anisotropy covariance structure

M3: Separable covariate-dependent ($z =$ geodetic depth) covariance structure

M4: Non-separable isotropic covariance structure

M5: Non-separable elliptical anisotropy covariance structure

M6: Non-separable, covariate-dependent ($z =$ geodetic depth) covariance structure

- $\gamma(s_t)$ assumed to be independent across time and common to both shores
- We allow for different local random effects for the North and South shores (Glémet and Rodríguez 2007; Schmidt et al. 2010b)

Outline
Motivation and aims
Distr. multiv. counts
Proposed model
Inference procedure
Results
Model comparison
Results
Conclusions
Main references

Fitted models

M1: Separable isotropic covariance structure

M2: Separable elliptical anisotropy covariance structure

M3: Separable covariate-dependent ($z =$ geodetic depth) covariance structure

M4: Non-separable isotropic covariance structure

M5: Non-separable elliptical anisotropy covariance structure

M6: Non-separable, covariate-dependent ($z =$ geodetic depth) covariance structure

- $\gamma(s_t)$ assumed to be independent across time and common to both shores
- We allow for different local random effects for the North and South shores (Glémet and Rodríguez 2007; Schmidt et al. 2010b)

Fitted models

M1: Separable isotropic covariance structure

M2: Separable elliptical anisotropy covariance structure

M3: Separable covariate-dependent ($z =$ geodetic depth) covariance structure

M4: Non-separable isotropic covariance structure

M5: Non-separable elliptical anisotropy covariance structure

M6: Non-separable, covariate-dependent ($z =$ geodetic depth) covariance structure

- $\gamma(s_t)$ assumed to be independent across time and common to both shores
- We allow for different local random effects for the North and South shores (Glémet and Rodríguez 2007; Schmidt et al. 2010b)

Outline
Motivation and aims
Distr. multiv. counts
Proposed model
Inference procedure
Results
Model comparison
Results
Conclusions
Main references

Fitted models

M1: Separable isotropic covariance structure

M2: Separable elliptical anisotropy covariance structure

M3: Separable covariate-dependent ($z =$ geodetic depth) covariance structure

M4: Non-separable isotropic covariance structure

M5: Non-separable elliptical anisotropy covariance structure

M6: Non-separable, covariate-dependent ($z =$ geodetic depth) covariance structure

- $\gamma(s_t)$ assumed to be independent across time and common to both shores
- We allow for different local random effects for the North and South shores (Glémet and Rodríguez 2007; Schmidt et al. 2010b)

Model comparison

Yellow perch

Model	p_D	DIC	EPD
M1	144.8	1039.0	2019063.4
M2	145.2	1038.6	2019061.1
M3	137.1	1031.3	2019023.1
M4	145.1	1040.0	2019030.6
M5	144.0	1036.4	2019067.2
M6	136.5	1032.1	2018988.0

Brown bullhead

Model	p_D	DIC	EPD
M1	109.4	626.2	133777.8
M2	110.1	628.3	133778.2
M3	101.1	621.3	133755.5
M4	111.9	633.7	133768.1
M5	112.6	635.1	133770.4
M6	101.1	622.9	133754.1

Golden shiner

Model	p_D	DIC	EPD
M1	106.0	590.1	246747.8
M2	106.0	591.7	246729.9
M3	98.6	585.4	246714.3
M4	103.2	592.1	246734.8
M5	105.5	596.2	246741.1
M6	99.6	590.6	246699.6

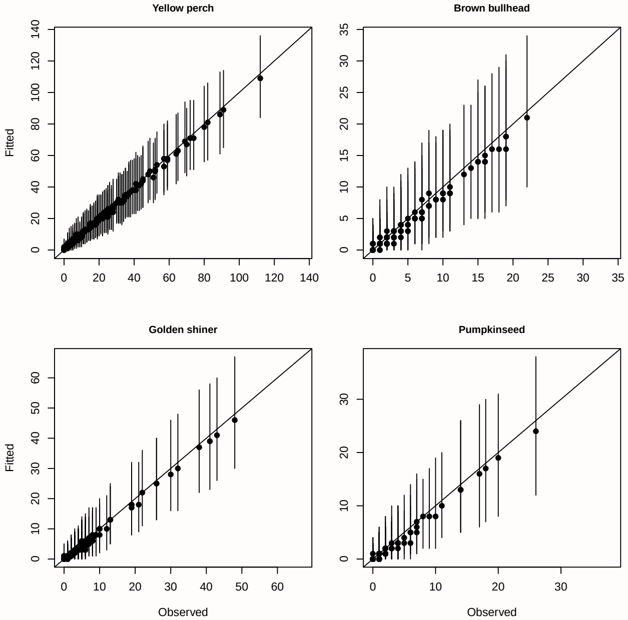
Pumpkinseed

Model	p_D	DIC	EPD
M1	81.9	432.4	54044.9
M2	81.4	429.8	54056.2
M3	75.6	428.9	54037.7
M4	82.9	434.0	54050.2
M5	82.8	434.7	54046.9
M6	75.7	432.7	54038.1

Table : Values of the effective number of parameters, p_D, DIC (Spiegelhalter et al. 2001), and EPD (Gelfand and Ghosh, 1998) for the six fitted models, by fish species.

Models including geodetic depth as a covariate in the correlation structure of the spatial effects generally fit better than those assuming isotropy or geometrical anisotropy

Outline
Motivation and aims
Distr. multiv. counts
Proposed model
Inference procedure
Results
Model comparison
Results
Conclusions
Main references



Posterior predictive distributions for counts of the four species under M6 showed agreement with observed counts

Outline

Motivation and aims

Distr. multiv. counts

Proposed model

Inference procedure

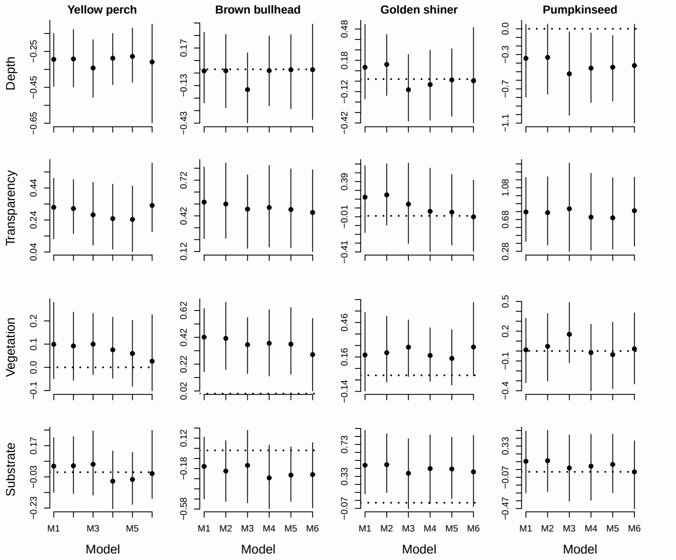
Results

Model comparison

Results

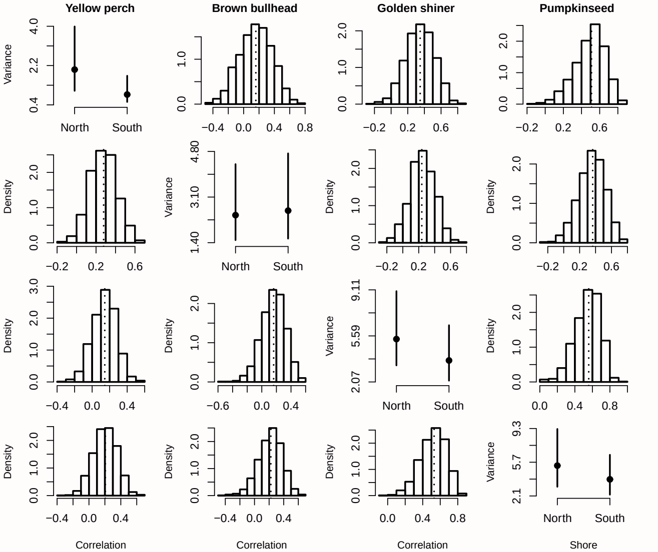
Conclusions

Main references



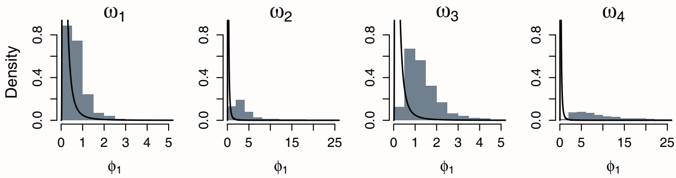
Water transparency had positive influence on the abundance of three of the four fish species, whereas water depth and vegetation each influenced the abundance of one fish species. Substrate composition had no apparent effect on fish abundances

Outline
Motivation and aims
Distr. multiv. counts
Proposed model
Inference procedure
Results
Model comparison
Results
Conclusions
Main references



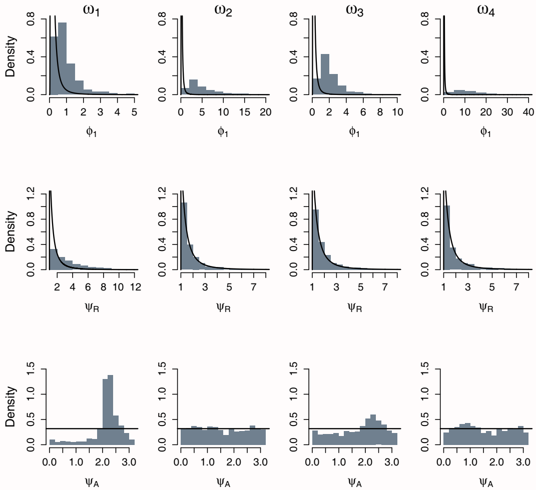
Variances and correlations tended to be greater in the North shore than in the South shore. The two strongest correlations, both in the North shore, were positive. Negative correlations, possibly indicative of competition between species, were not apparent.

Outline
Motivation and aims
Distr. multiv. counts
Proposed model
Inference procedure
Results
Model comparison
Results
Conclusions
Main references



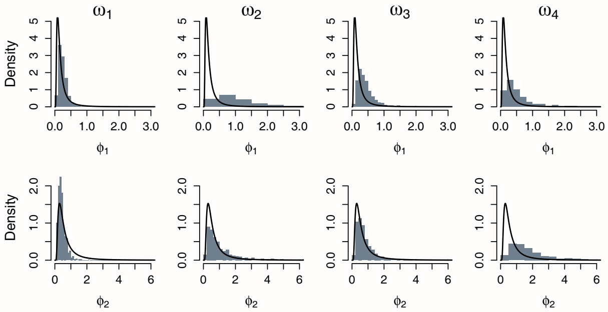
For all non-separable models, the decay parameter ϕ_1 showed well-defined information gain and marked differences among component spatial processes $\omega_k(.)$.

Outline
Motivation and aims
Distr. multiv. counts
Proposed model
Inference procedure
Results
Model comparison
Results
Conclusions
Main references



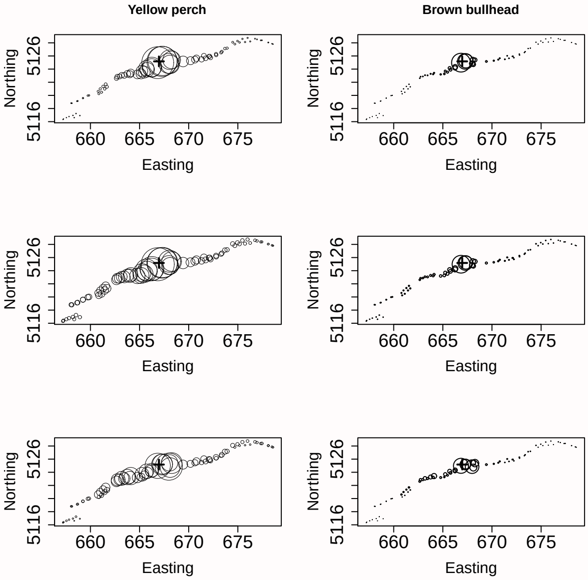
M5 provided strong evidence of anisotropy associated with spatial process $\omega_1(.)$, but little deviation from the priors for components $\omega_2(.)$, $\omega_3(.)$, and $\omega_4(.)$, indicating a single anisotropic pattern shared across species and suggesting that the spatial component of M5 may be overparametrized.

Outline
Motivation and aims
Distr. multiv. counts
Proposed model
Inference procedure
Results
Model comparison
Results
Conclusions
Main references



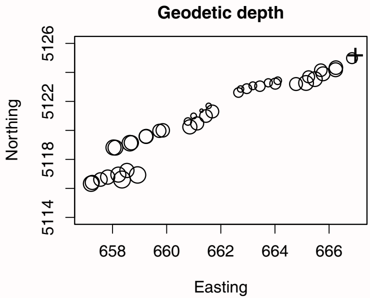
Similar to M5, the posterior distribution of ϕ_2 under M6 provides strong evidence of anisotropy. However, in contrast to M5, the gain in information reflected in differences between priors and posteriors is distributed across components, indicating that species do not share a common anisotropic pattern.

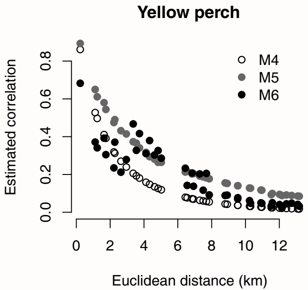
Outline
Motivation and aims
Distr. multiv. counts
Proposed model
Inference procedure
Results
Model comparison
Results
Conclusions
Main references

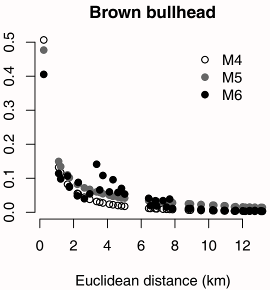


Decay of correlation under M6 had directionality generally similar to that of M5, but was further modified by geodetic lake depth.

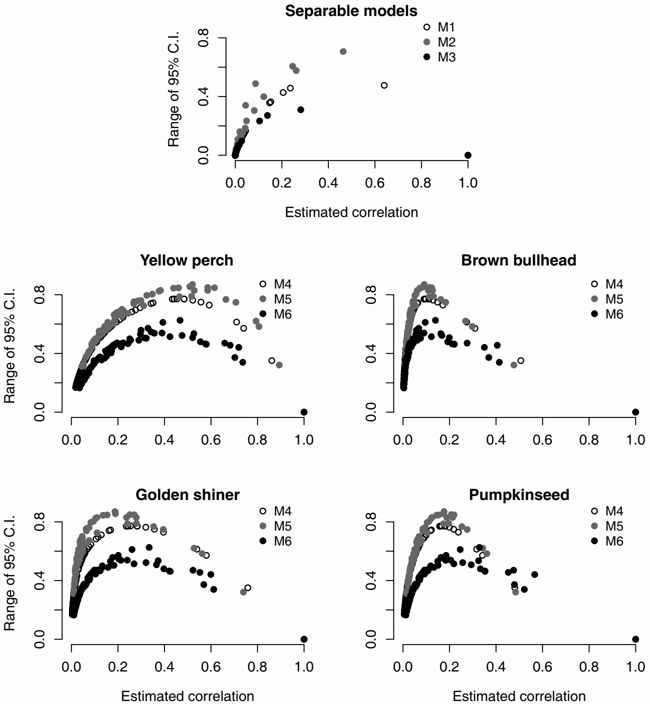
Outline
Motivation and aims
Distr. multiv. counts
Proposed model
Inference procedure
Results
Model comparison
Results
Conclusions
Main references







Outline
Motivation and aims
Distr. multiv. counts
Proposed model
Inference procedure
Results
Model comparison
Results
Conclusions
Main references



Alex Schmidt
IM-UFRJ
Brazil

PASI 2014
Multivariate Spatial Stats
Búzios, Brazil, 2014

Outline
Motivation and aims
Distr. multiv. counts
Proposed model
Inference procedure
Results
Model comparison
Results
Conclusions
Main references

Conclusions

- Our model accounts for overdispersion and both positive and negative covariances (among species and across space)
- The LMC provided flexible covariance structures for the mixing component
- Anisotropic spatial effects improved fit (DIC and EPD)
- Including information on geodetic lake depth in the spatial covariance structure of the spatial process provided a flexible, yet relatively simple, means of capturing anisotropy along the shorelines of the lake

Alex Schmidt
IM-UFRJ
Brazil

PASI 2014
Multivariate Spatial Stats
Búzios, Brazil, 2014

Outline
Motivation and aims
Distr. multiv. counts
Proposed model
Inference procedure
Results
Model comparison
Results
Conclusions
Main references

Main references

- Aitchison, J. and Ho, C. H. (1989). The multivariate Poisson-log normal distribution. *Biometrika*, **76**, 643–653.
- Chib, S. and Winkelmann, R. (2001). Markov chain Monte Carlo analysis of correlated count data. *J. Business & Economic Statistics*, **19**, 428–435.
- Gelfand, A. E., Schmidt, A. M., Banerjee, S. and Sirmans, C. F. (2004). Nonstationary multivariate process modeling through spatially varying coregionalization (with discussion), *Test*, **13**, 263–312.
- Glémet, H. and Rodríguez, M. A. (2007). Short-term growth (RNA/DNA ratio) of yellow perch (*Perca flavescens*) in relation to environmental influences and spatio-temporal variation in a shallow fluvial lake. *Can. J. Fish. Aquat. Sci.*, **64**, 1646–1655.
- Schmidt, A. M. and Gelfand, A. E. (2003). A Bayesian coregionalization model for multivariate pollutant data. *J. Geophysics Res. - Atmospheres*, **108**, STS10.1-STS10.8.
- Schmidt, A. M., Guttorp, P. and O'Hagan, A. (2011). Considering covariates in the covariance structure of spatial processes. *Environmetrics*, **22**, 487-500.
- Schmidt, A. M., Rodríguez, M. A. and Capistrano, E. S. (2012). Accounting for latent spatio-temporal structure in animal abundance models. *Technical Report Depto de Métodos Estatísticos, IM-UFRJ, Brazil*.

Outline

Motivation and aims

Distr. multiv. counts

Proposed model

Inference procedure

Results

Model comparison

Results

Conclusions

Main references