Calder + Risser's Thoughts on Fitting PC Models

- The Process Convolution Model (as in the Practicum)...

  $\rightarrow$ *written hierarchically*

$$\boldsymbol{Z}|\boldsymbol{Y}, \tau^2 \sim \mathrm{N}(\boldsymbol{Y}, \tau^2 \boldsymbol{I})$$

$$\boldsymbol{Y}|\boldsymbol{\Psi}_1, \ldots, \boldsymbol{\Psi}_n \sim \mathrm{GP}(\mathbf{0}, \boldsymbol{\Sigma}^Y(\boldsymbol{\Psi}_1, \ldots, \boldsymbol{\Psi}_n))$$

$$\text{Kernel Parameters:} \begin{cases} \psi_1^1, \ldots, \psi_n^1 | m_1, v_1, r_1 \sim \mathrm{GP}(m_1 \mathbf{1}, \boldsymbol{\Sigma}^\psi(v_1, r_1)) \\ \psi_1^2, \ldots, \psi_n^2 | m_2, v_2, r_2 \sim \mathrm{GP}(m_2 \mathbf{1}, \boldsymbol{\Sigma}^\psi(v_2, r_2)) \\ \psi_1^3, \ldots, \psi_n^3 | m_3, v_3, r_3 \sim \mathrm{GP}(m_3 \mathbf{1}, \boldsymbol{\Sigma}^\psi(v_3, r_3)) \end{cases}$$

$$\tau^2, \boldsymbol{m}, \boldsymbol{v}, \boldsymbol{r} \sim \text{something}$$

where $\boldsymbol{\Sigma}^Y(\cdot)$ and $\boldsymbol{\Sigma}^\psi(\cdot)$ are matrix-valued functions of their inputs

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- Integrating out $\boldsymbol{Y}$, the unknown parameters in the model are

$$\underbrace{\boldsymbol{\Psi}_1}_{3\times 1}, \ldots, \underbrace{\boldsymbol{\Psi}_n}_{3\times 1}, \tau^2, \boldsymbol{m}, \boldsymbol{v}, \boldsymbol{r}$$

  for a total of

$$3n + 1 + 3 + 3 + 3 = 2038$$

  (highly dependent!) parameters in the simulated gridded data

- Since we can't integrate out the $\boldsymbol{\Psi}$s analytically, MCMC will likely not work $\rightarrow$ the posterior distribution is highly structured and the dimension of the parameter space is large

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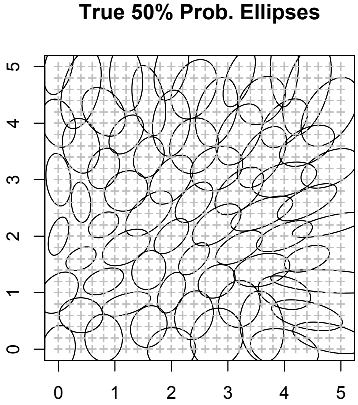
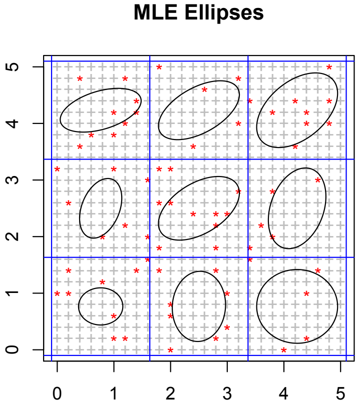
- The good news... there appears to be information about the **Ψ**s in a single realization from the model





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- Our thoughts:

  1. the GP priors on the elements of the $\boldsymbol{\Psi}$ vectors makes the model overly parameterized
  2. the $\boldsymbol{\Psi}$s instead should perhaps be deterministic functions of a few unknown parameters
  3. the model itself isn't bad, but more work is needed on prior specification & collapsing a layer of the hierarchy