Modelling spatial extremes with the SpatialExtremes package

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Rationale for the `SpatialExtremes` package

> "The aim of the `SpatialExtremes` package is to provide tools for the areal modelling of extreme events. The modelling strategies heavily rely on the extreme value theory and in particular block maxima techniques—unless explicitly stated."

As a consequence, most often

- the data used by the package have to be extreme—do not pass daily values for instance;
- the marginal distribution family is fixed, i.e., the generalized extreme value distribution family, but you have hands on how within this family parameters change in space;
- the process family is fixed, i.e., max-stable processes, but you have hands on which type of max-stable processes to use.

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1. Data and descriptive analysis

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Before introducing more advanced stuff, let's talk about data format. It is pretty simple

Observations A numeric matrix such that each row is one realization of the spatial field—or if you prefer one column per site;

Coordinates A numeric matrix such that each row is the coordinates of one site—or if you prefer the first column is for instance the longitude of all sites, the second one latitude, …

```
> data
      Valkenburg  Ijmuiden  De Kooy  ...
 1971        278        NA      360  ...
 1972        334        NA      376  ...
 1973        376        NA      365  ...
 1974        314        NA      304  ...
 1975        278        NA      278  ...
 1976        350        NA      345  ...
 1977        324        NA      298  ...
 1978        298        NA      329  ...
 1979        252        NA      298  ...
 ...
```

```
> coord
                    lon      lat
 Valkenburg       4.419   52.165
 Ijmuiden         4.575   52.463
 De Kooy          4.785   52.924
 Schiphol         4.774   52.301
 Vlieland         4.942   53.255
 Berkhout         4.979   52.644
 Hoorn            5.346   53.393
 De Bilt          5.177   52.101
 ...
```

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In addition to the storage of observations and coordinates, you might want to use additional covariates. The latter can be of two types

Spatial A numeric matrix such that each column corresponds to one spatial covariate such as elevation, urban/rural, ...

Temporal A numeric matrix such that each column corresponds to one temporal covariate such as time, annual mean temperature, ...

```
> spat.cov
                alt
Valkenburg     -0.2
Ijmuiden        4.4
De Kooy         0.5
Schiphol       -4.4
Vlieland        0.9
Berkhout       -2.5
Hoorn           0.5
De Bilt         2.0
...
```

```
> temp.cov
            nao
1971       1.87
1972       1.57
1973      -0.20
1974      -0.95
1975      -0.46
1976       2.34
1977      -0.49
1978       0.70
1979       1.11
...
```

✍ *It is always a good idea to name your columns and rows.*

Inspecting data

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- As usual, you first have to scrutinize your data (weird values, encoding of missing values, check out factors, ...). But you're used to that, aren't you?
- We focus on extremes, so you may wonder
  - are my data extremes, i.e., block maxima?
  - is my block size relevant?
  - what about seasonality? Refine the block or use temporal covariate?
- You might want to check that the generalized extreme value family is sensible for your data—the `evd` package + a few lines of code will do the job for you (homework)
- This will generally be OK, but now you have to go a bit further by analyzing
  - the spatial dependence ;
  - and the presence / absence of any spatial trends.

Spatial dependence

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- Essentially you want to check if your data exhibit any (spatial) dependence. If not why would you bother with spatial models?
- The most convenient way to do this is through the F-madogram and its connection with the extremal coefficient:

$$\nu_F(h) = \frac{1}{2}\mathbb{E}[|F\{Z(o)\} - F\{Z(h)\}|], \qquad \theta(h) = \frac{1+2\nu_F(h)}{1-2\nu_F(h)}.$$

- Recall that $1 \leq \theta(h) \leq 2$ where complete dependence iff $\theta(h) = 1$ and independence iff $\theta(h) = 2$.
- The `fmadogram` function will estimate (empirically) the pairwise extremal coefficent from the F–madogram.

The `fmadogram` function

- Run the file `fmadogram.R`. You should get the figure below. Any questions?

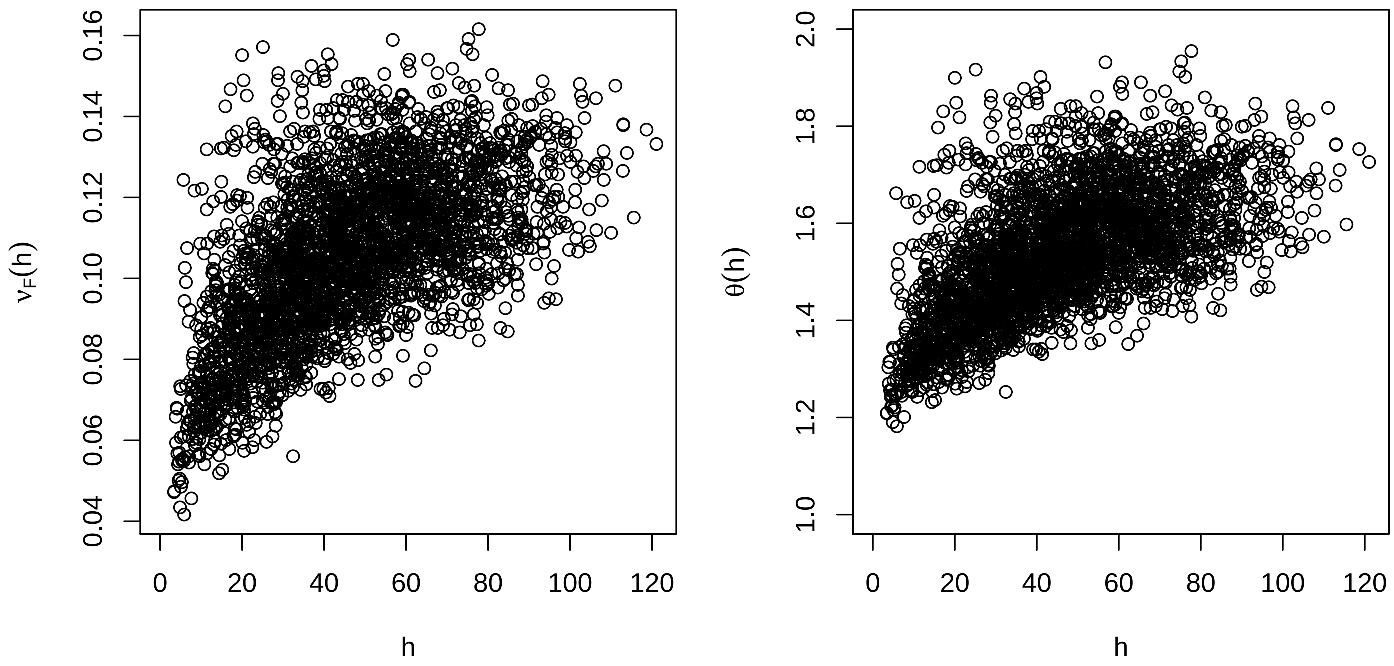

Figure 1: *Use of the `fmadogram` function to assess the spatial dependence.*

The `fmadogram` function

- Run the file `fmadogram.R`. You should get the figure below. Any questions?
- No? What's wrong?

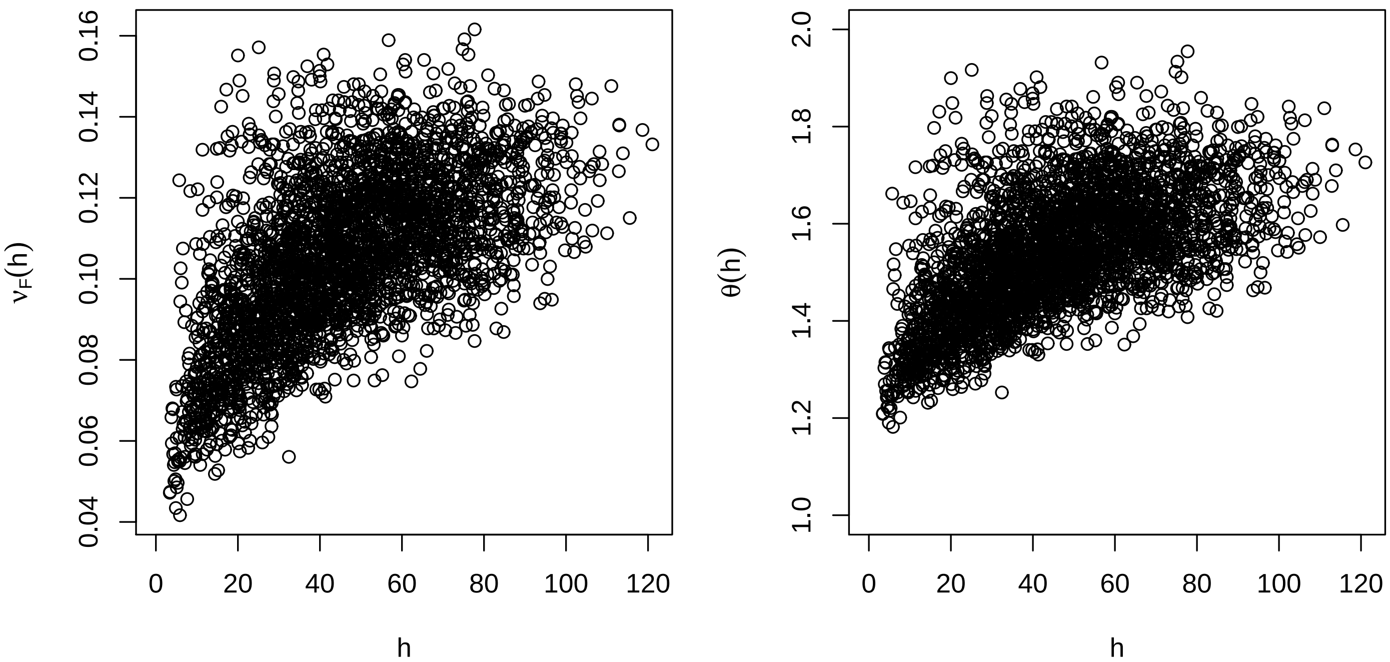

Figure 1: *Use of the `fmadogram` function to assess the spatial dependence.*

The `fmadogram` function

- Run the file `fmadogram.R`. You should get the figure below. Any questions?
- No? What's wrong?
- You can also use a binned version with `n.bins = 300`...

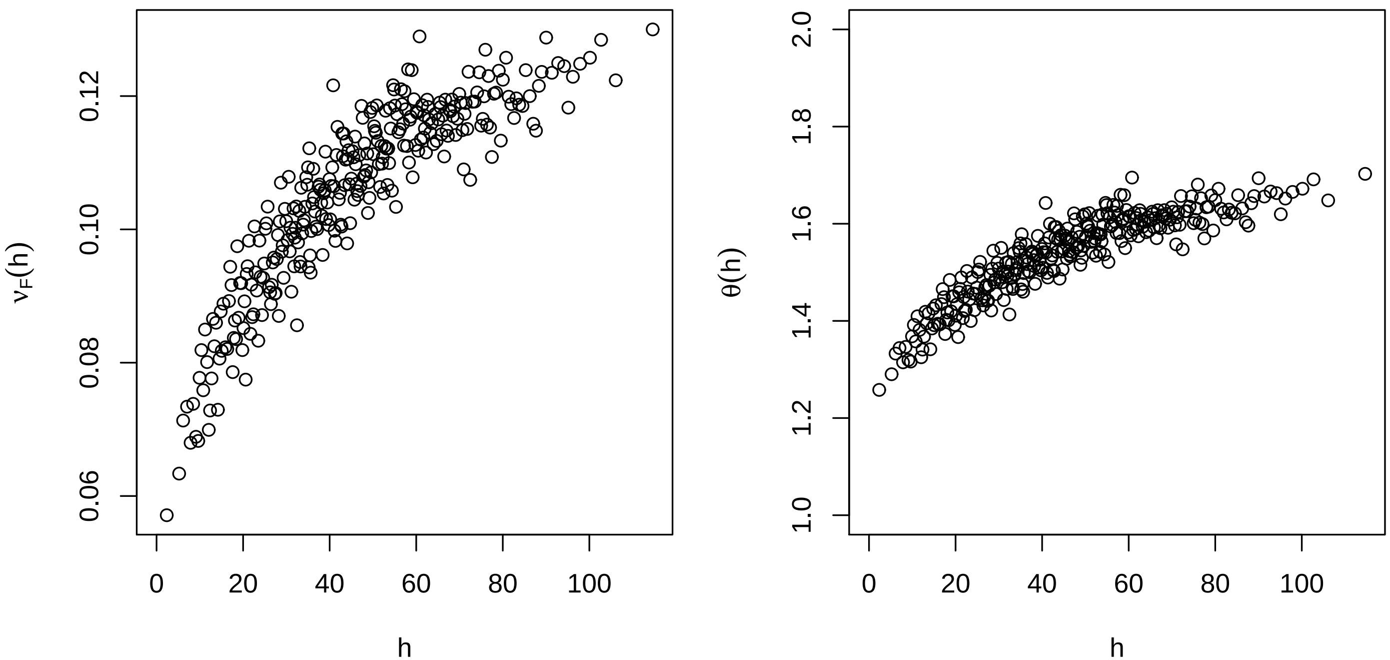

Figure 1: *Use of the `fmadogram` function to assess the spatial dependence.*

Spatial trends

- We can do a symbol plot but the package doesn't have (yet?) a function for this—mainly because it's application specific.
- Examples at `SpatialTrends.R` and `SpatialTrends2.R`

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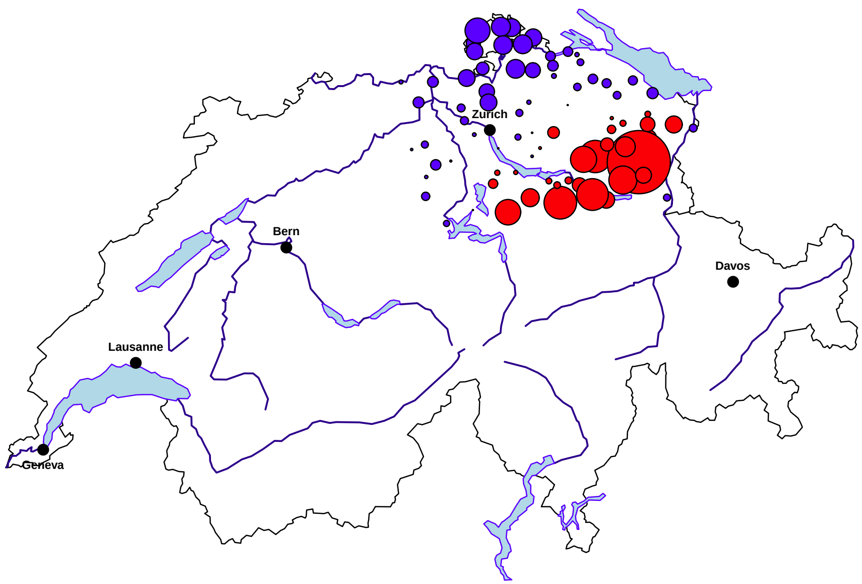

Figure 2: *Symbol plot for the swiss precipitation data.*

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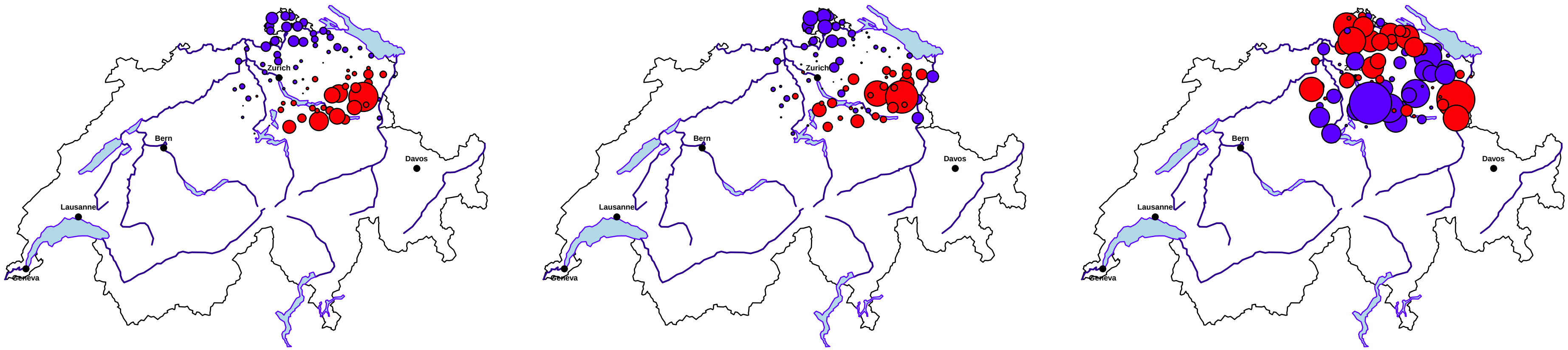

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Spatial trends

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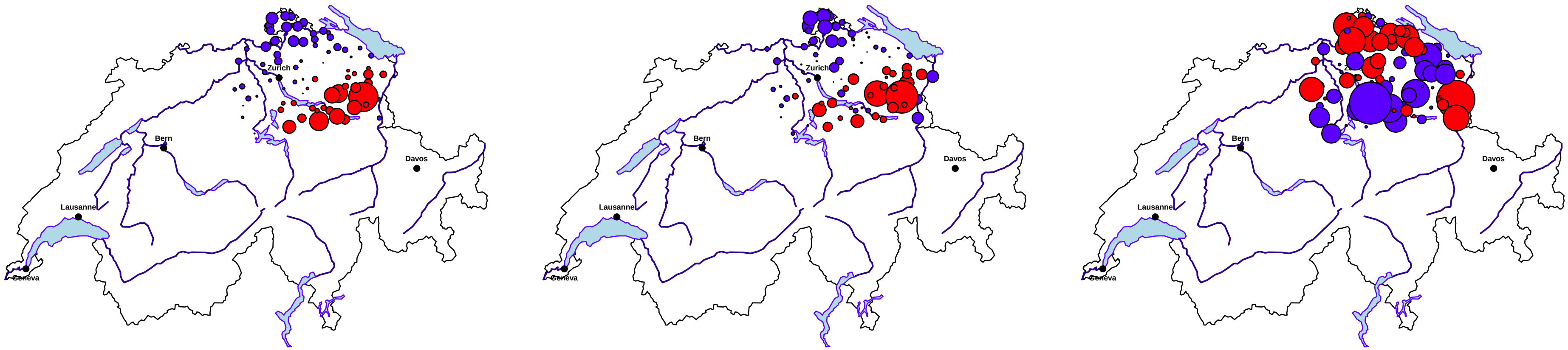

Figure 2: *Symbol plot for the swiss precipitation data.*

✍ *When exporting figures into eps/pdf, always pay attention to the aspect ratio.*

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- The data exhibit some spatial dependence. The extremal coefficient is around 1.7 for a separation lag of 100km—extremes are still not independent but close to.
- There's a clear north-west / south-east gradient in the intensities of rainfall storms.
- In conclusion it makes sense to use max-stable models whose marginal parameters are not constant across space.
- More specifically, we have
  - a clear north-west / south-east gradient for the location and scale parameters;
  - no clear pattern for the shape parameter.

Homework

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- Have a look at the temperature and the wind gust data;
- Do a descriptive analysis for these two data sets.

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2. Simple max-stable processes

Max-stable models

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- In this section we focus only on the spatial dependence and so assume that the margins are known and unit Fréchet—this is a standard choice in extreme value theory.
- From the spectral characterization (Dan's lecture)

$$Z(x) = \max_{i \geq 1} \zeta_i Y_i(x), \qquad x \in \mathcal{X} \subset \mathbb{R}^d,$$

we can propose several parametric models for spatial extremes. Hence by letting Y to be

Gaussian densities with random displacements we get the Smith process;

Gaussian we get the Schlather process;

Log-normal (with a drift) we get the Brown–Resnick process;

Gaussian but elevated to some power we get the Extremal-t process.

Dependence parameters

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Smith Elements of the covariance matrix appearing in the Gaussian densities;

Schlather Parameters of the correlation function;

Brown–Resnick Parameters of the semi-variogram;

Extremal–*t* Parameters of the correlation function and degrees of freedom.

- Since the margins are fixed, we only need to get estimates for the dependence parameters.
- How can we do that?

Least squares (`leastSquares.R`)

$$\arg\min_{\psi\in\Psi} \sum_{1\leq i<j\leq k} \left\{\theta(x_j - x_j;\psi) - \hat{\theta}(x_i - x_j)\right\}^2,$$

where $\theta(\cdot;\psi)$ is the extremal coefficient obtained from the max-stable model with dependence parameters set to ψ and $\hat{\theta}(\cdot)$ is any empirical estimates of the extremal coefficient, e.g., F–madogram based.

```
> M0
        Estimator: Least Squares
            Model: Schlather
         Weighted: TRUE
  Objective Value: 3592.429
Covariance Family: Whittle-Matern

Estimates
  Marginal Parameters:
  Assuming unit Frechet.

  Dependence Parameters:
  range   smooth
54.3239   0.4026

Optimization Information
  Convergence: successful
  Function Evaluations: 61
```

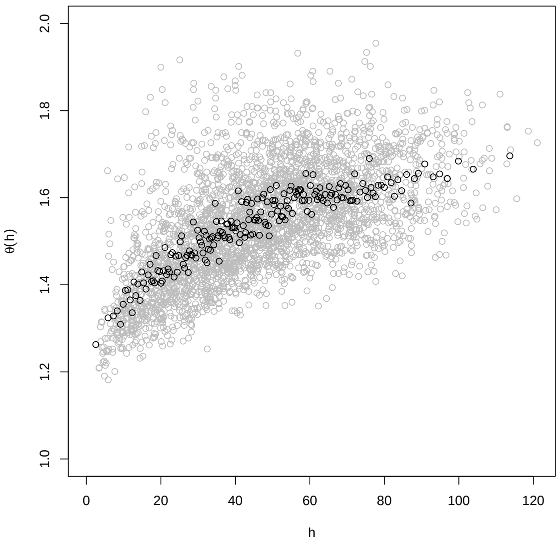

Figure 3: *Fitting simple max-stable processes from least squares.*

$$\underset{\psi\in\Psi}{\arg\min} \sum_{1\leq i<j\leq k} \left\{\theta(x_j - x_j;\psi) - \hat{\theta}(x_i - x_j)\right\}^2,$$

where $\theta(\cdot;\psi)$ is the extremal coefficient obtained from the max-stable model with dependence parameters set to ψ and $\hat{\theta}(\cdot)$ is any empirical estimates of the extremal coefficient, e.g., F–madogram based.

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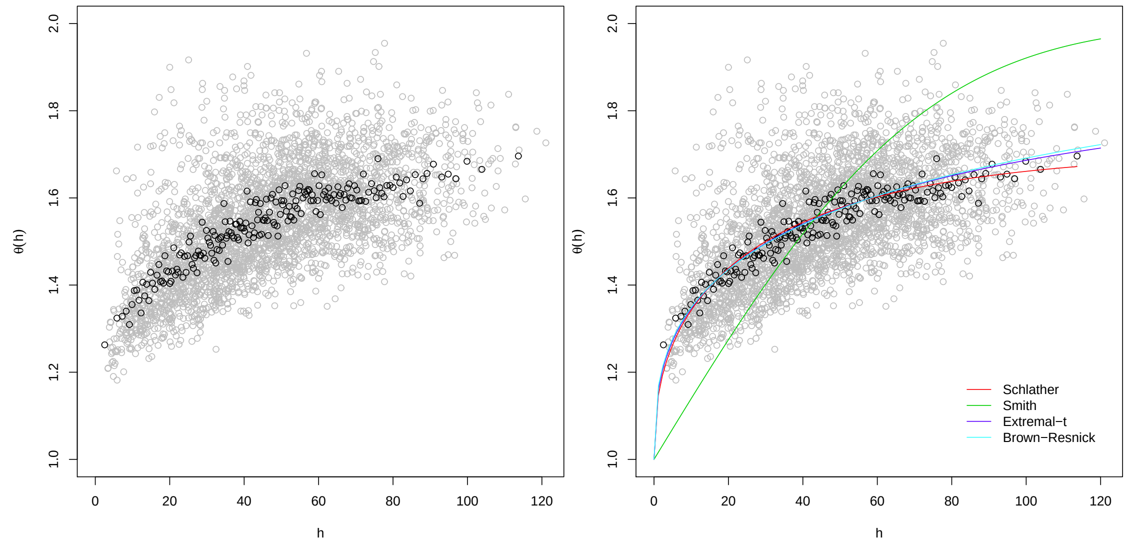

Figure 3: *Fitting simple max-stable processes from least squares.*

$$\arg\max_{\psi \in \Psi} \sum_{\ell=1}^{n} \sum_{1 \leq i < j \leq k} \log f\{z_\ell(x_i), z_\ell(x_j); \psi\},$$

where $f(\cdot, \cdot; \psi)$ is the bivariate density of the considered max-stable model.

```
        Estimator: MPLE
            Model: Schlather
         Weighted: FALSE
   Pair. Deviance: 1136863
              TIC: 1137456
Covariance Family: Whittle-Matern

Estimates
  Marginal Parameters:
  Assuming unit Frechet.

  Dependence Parameters:
  range   smooth
50.1976   0.3713

Standard Errors
  range   smooth
20.7085   0.0789

Asymptotic Variance Covariance
        range       smooth
range   428.841018   -1.570081
smooth   -1.570081    0.006225
Optimization Information
  Convergence: successful
  Function Evaluations: 67
```

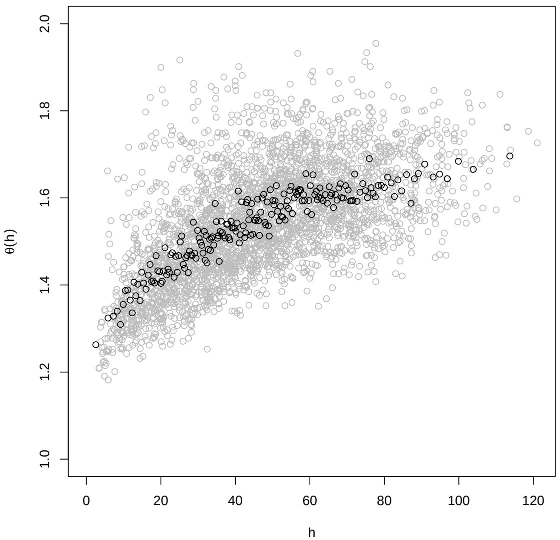

Figure 4: *Fitting simple max-stable processes maximizing pairwise likelihood.*

Pairwise likelihood (`pairwiseLlik.R`)

$$\arg\max_{\psi\in\Psi} \sum_{\ell=1}^{n} \sum_{1\leq i<j\leq k} \log f\{z_\ell(x_i), z_\ell(x_j); \psi\},$$

where $f(\cdot,\cdot;\psi)$ is the bivariate density of the considered max-stable model.

```
         Estimator: MPLE
             Model: Schlather
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    Pair. Deviance: 1136863
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Estimates
  Marginal Parameters:
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  range   smooth
50.1976   0.3713

Standard Errors
  range   smooth
20.7085   0.0789

Asymptotic Variance Covariance
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range   428.841018  -1.570081
smooth   -1.570081   0.006225
Optimization Information
  Convergence: successful
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```

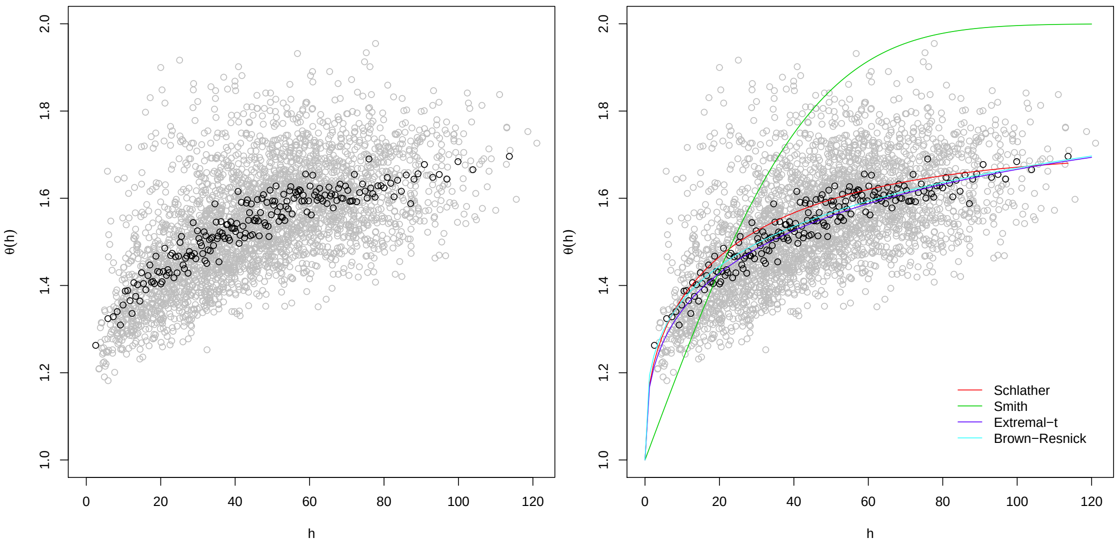

Figure 4: *Fitting simple max-stable processes maximizing pairwise likelihood.*

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- The advantage of the pairwise likelihood estimator over the least squares one is that you can do model selection.
- For instance one can use the TIC, Takeuchi Information Criterion or sometimes known as CLIC, Composite Likelihood Information Criterion,

$$\mathrm{TIC} = 2\ell_{\text{pairwise}}(\hat{\psi}) - 2\mathrm{tr}\{J(\hat{\psi}) H^{-1}(\hat{\psi})\},$$

$$H(\hat{\psi}) = \mathbb{E}\{\nabla^2 \ell_{\text{pairwise}}(Y;\hat{\psi})\}, \; J(\hat{\psi}) = \mathrm{Var}\{\nabla \ell_{\text{pairwise}}(Y;\hat{\psi})\}.$$

- From our previous fitted models, we get

```
> TIC(M0,M1,M2,M3)
     M2      M3      M0      M1
1133668 1134829 1137456 1159784
```

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- Once you have fitted a suitable model, you usually want to simulate from it.
- Simulation from max-stable models is rather complex, recall that

$$Z(x) = \max_{i \geq 1} \zeta_i Y_i(x), \qquad x \in \mathcal{X}.$$

```
sim <- rmaxstab(n.obs, cbind(x, y), "twhitmat", DoF = 4,
+               nugget = 0, range = 3, smooth = 1)
> sim
          [,1]      [,2]      [,3]      [,4]      [,5]
 [1,]  3.8048914  0.4767980 6.3613989 1.4548317  1.0433912
 [2,]  1.2200332  0.6711422 0.8078701 2.0928629  0.7537061
 [3,]  0.5466466  2.0498561 4.8852572 2.3497976  0.6857268
 [4,]  ...
```

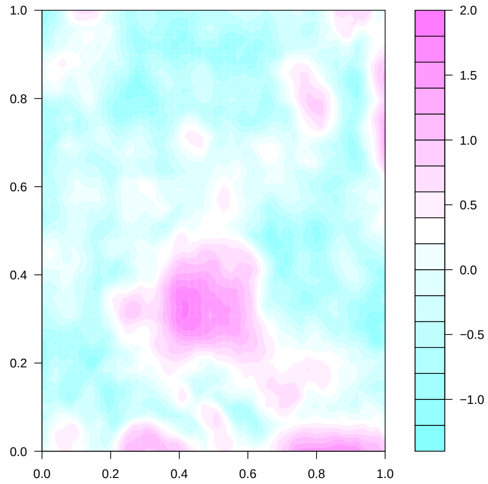

Figure 5: *One simulation on a 50 x 50 grid from the extremal–t model. (log scale)*

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- The Smith model is clearly not a sensible model for our data—because of its linear behaviour near the origin;
- Schlather, Brown–Resnick and Extremal–t seems relevant;
- According to the TIC, the Extremal–t should be preferred.

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Least squares

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- Perform simulations from various max-stable processes on scattered and lattice locations;
- Fit a Schlather model with a powered exponential correlation function;
- Why do we always set `nugget = 0`?
- Try to put weights within the pairwise likelihood.

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Spatial GEV

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- Alright we are able to handle the spatial dependence, but we assume that our data have unit Fréchet margins. This is not realistic at all!
- Fortunately, if $Y \sim \text{GEV}(\mu, \sigma, \xi)$ then

$$Z = \left(1 + \xi \frac{Y - \mu}{\sigma}\right)^{1/\xi}$$

is a unit Fréchet random variable.

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- Alright we are able to handle the spatial dependence, but we assume that our data have unit Fréchet margins. This is not realistic at all!
- Fortunately, if $Y \sim \text{GEV}(\mu, \sigma, \xi)$ then

$$Z = \left(1 + \xi \frac{Y - \mu}{\sigma}\right)^{1/\xi}$$

  is a unit Fréchet random variable.
- And since we are extreme value and spatial guys

$$Z(x) = \left\{1 + \xi(x) \frac{Y(x) - \mu(x)}{\sigma(x)}\right\}^{1/\xi(x)}, \qquad x \in \mathcal{X},$$

  is a simple max-stable process.
- Hence we can use the pairwise likelihood estimator as before—up to an additional Jacobian term.

Omitting the spatial dependence

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- With simple max-stable models, we omitted the marginal parameters.
- Here we will omit the spatial dependence for a while and consider locations as being mutually independent, i.e., use independence likelihood

$$\arg\max_{\psi \in \Psi} \sum_{i=1}^{k} \ell_{\text{GEV}}\{y(x_i);\psi\}.$$

- This is a kind of "*spatial GEV*" where ψ is a vector of marginal parameters.

Defining trend surfaces

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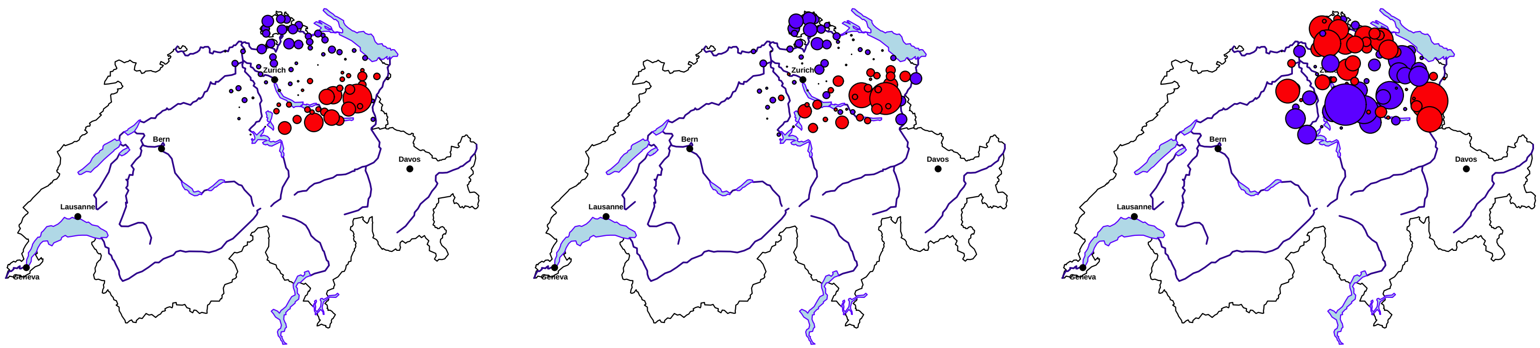

Figure 6: *Symbol plot for the swiss precipitation data.*

This suggest that

$$
\begin{aligned}
\mu(x) &= \beta_{0,\mu} + \beta_{1,\mu}\mathrm{lon}(x) + \beta_{2,\mu}\mathrm{lat}(x) + \beta_{3,\mu}\mathrm{lon}(x) \times \mathrm{lat}(x),\\
\sigma(x) &= \beta_{0,\sigma} + \beta_{1,\sigma}\mathrm{lon}(x) + \beta_{2,\sigma}\mathrm{lat}(x) + \beta_{3,\sigma}\mathrm{lon}(x) \times \mathrm{lat}(x),\\
\xi(x) &= \beta_{0,\xi},
\end{aligned}
$$

or equivalently with the `R` language

```
loc.form <- scale.form <- y ~ lon * lat; shape.form <- y ~ 1
```

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```
        Model: Spatial GEV model
     Deviance: 29303.81
          TIC: 29499.38

      Location Parameters:
locCoeff1  locCoeff2  locCoeff3  locCoeff4
   27.132      1.846     -3.656     -1.080
         Scale Parameters:
scaleCoeff1  scaleCoeff2  scaleCoeff3  scaleCoeff4
     9.7850       0.7023      -1.0858      -0.5531
         Shape Parameters:
shapeCoeff1
     0.1572

Standard Errors
  locCoeff1    locCoeff2    locCoeff3    locCoeff4  scaleCoeff1  scaleCoeff2
    1.13326      0.34864      0.45216      0.38361      0.76484      0.28446
scaleCoeff3  scaleCoeff4  shapeCoeff1
    0.31267      0.27566      0.05878

Asymptotic Variance Covariance
             locCoeff1   locCoeff2   locCoeff3   locCoeff4   scaleCoeff1
locCoeff1     1.2842711   0.1131400  -0.1740921  -0.0729564   0.6570988
locCoeff2     0.1131400   0.1215498  -0.0623759   0.0149596   0.0521630
locCoeff3    -0.1740921  -0.0623759   0.2044448   0.0576622  -0.1086629
locCoeff4    -0.0729564   0.0149596   0.0576622   0.1471593  -0.0346376
scaleCoeff1   0.6570988   0.0521630  -0.1086629  -0.0346376   0.5849729
...

Optimization Information
  Convergence: successful
  Function Evaluations: 2135
```

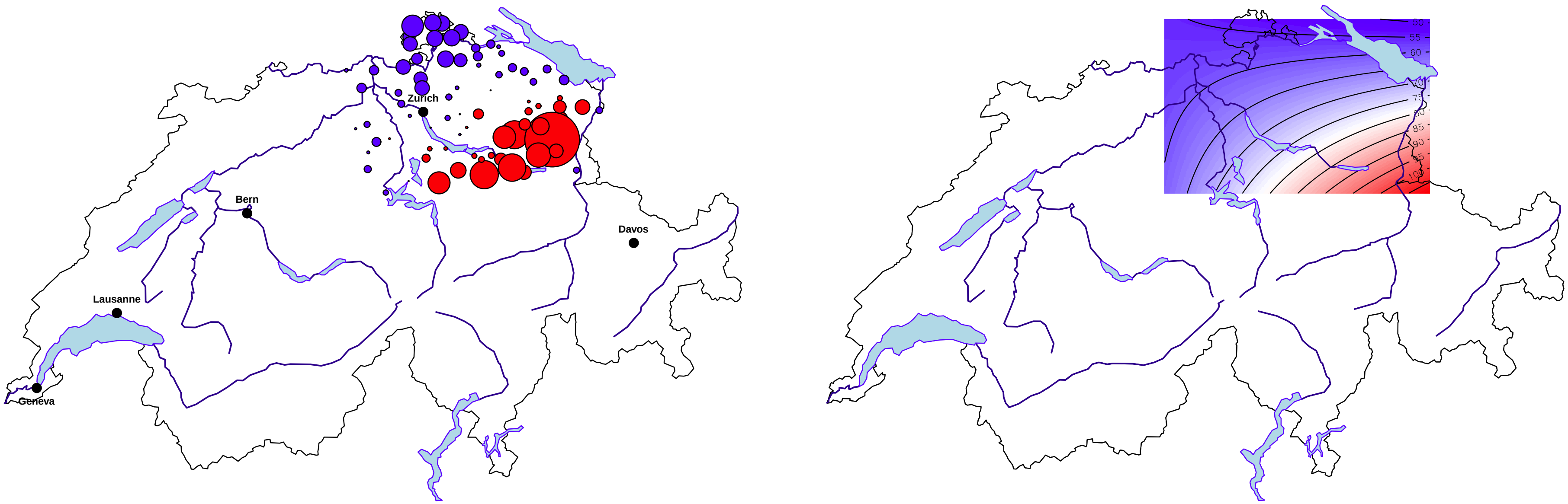

Figure 7: *Left: symbol plot. Right: Prediction of the pointwise 25-year return levels from a fitted spatial GEV model.*

Get predictions (`predictionSpatialGEV.R`)

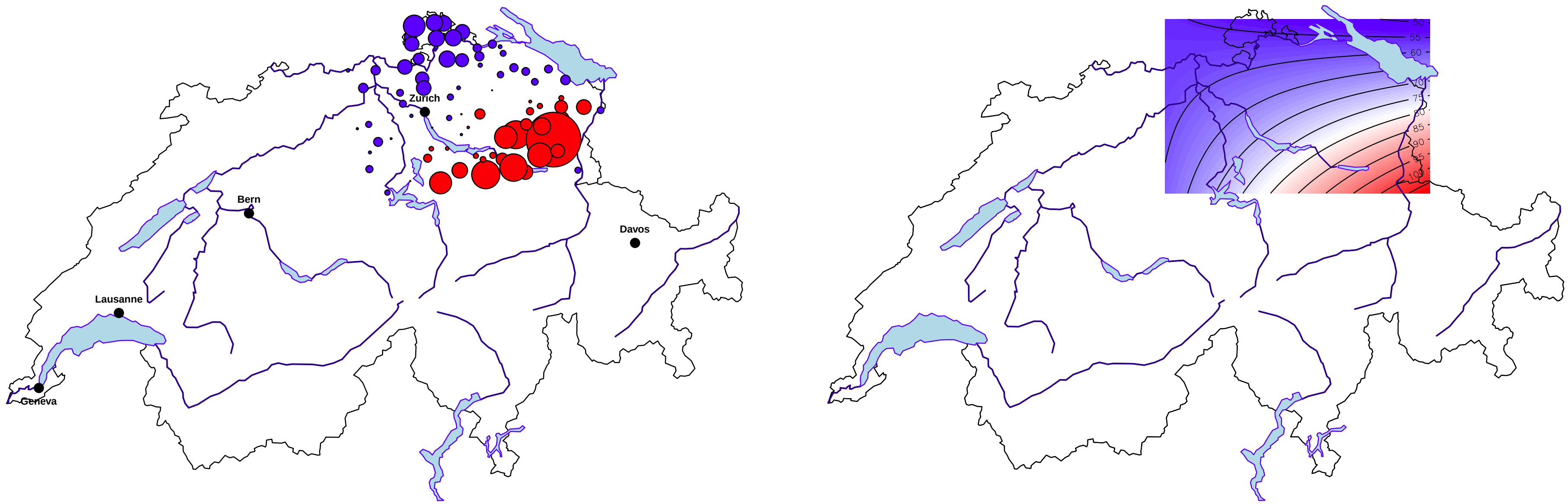

Figure 7: *Left: symbol plot. Right: Prediction of the pointwise 25-year return levels from a fitted spatial GEV model.*

- But don't we forget something???

Get predictions (`predictionSpatialGEV.R`)

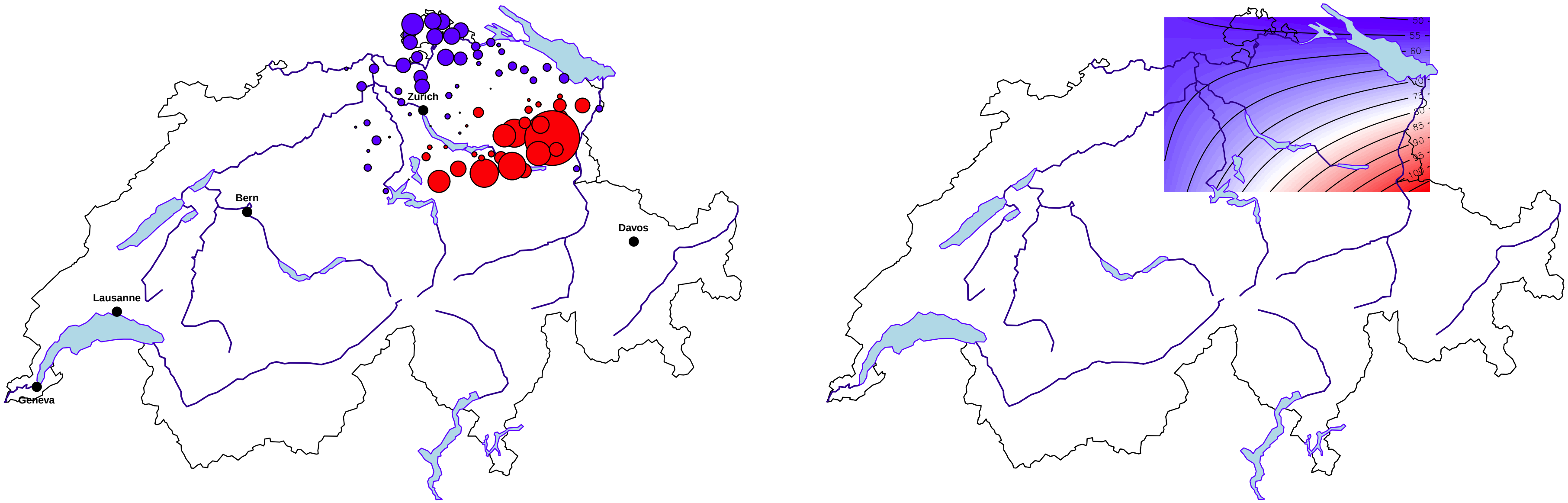

Figure 7: *Left: symbol plot. Right: Prediction of the pointwise 25-year return levels from a fitted spatial GEV model.*

- But don't we forget something???
- Model selection?

1. Data and descriptive analysis
2. Simple max-stable processes
3. Trends surfaces
Spatial GEV
Prediction #1
▷ Model selection #2
Debrief #3
Homework
4. General max-stable processes
5. Conclusion

- Typically here we would like to test if a given covariate is required or not
- Hence we're dealing with nested model for which composite likelihood ratio test are especially suited

$$2\{\ell_{\text{composite}}(\hat{\psi}) - \ell_{\text{composite}}(\hat{\phi}_{\lambda_0}, \lambda_0)\} \longrightarrow \sum_{j=1}^{p} \lambda_j X_i, \qquad n \to \infty.$$

```
Eigenvalue(s):
0.06
0.04

Analysis of Variance Table
   MDf Deviance Df  Chisq Pr(> sum lambda Chisq)
M2   7    29329
M0   9    29304  2 24.924              < 2.2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

✍ *Always check that your models are nested. The code won't do that for you!*

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- Based on the `spatial GEV` model, we identify what seems to be relevant trend surfaces for the marginal parameters:

$$\mu(x) = \beta_{0,\mu} + \beta_{1,\mu}\mathrm{lon}(x) + \beta_{2,\mu}\mathrm{lat}(x),$$
$$\sigma(x) = \beta_{0,\sigma} + \beta_{1,\sigma}\mathrm{lon}(x) + \beta_{2,\sigma}\mathrm{lat}(x),$$
$$\xi(x) = \beta_{0,\xi},$$

Homework

1. Data and descriptive analysis
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- Produce a figure similar to Figure 7 with our best model;
- Fit a *spatial GEV* model where elevation appears;
- Is this model appropriate?

1. Data and descriptive analysis
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3. Trends surfaces
▷ 4. General max-stable processes
Fitting
Model checking
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Homework
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4. General max-stable processes

Fitting a max-stable process with trend surfaces

- Now it's time to combine everything, i.e., trend surfaces + dependence.
- The syntax won't be a big surprise

```
M0 <- fitmaxstab(rain, coord[,1:2], "twhitmat", nugget = 0, loc.form, scale.form, shape.form)

        Estimator: MPLE
            Model: Extremal-t
         Weighted: FALSE
   Pair. Deviance: 2239596
              TIC: 2251000
Covariance Family: Whittle-Matern

Estimates
  Marginal Parameters:
    Location Parameters:
locCoeff1  locCoeff2  locCoeff3
 20.65202    0.06473   -0.15630
       Scale Parameters:
scaleCoeff1  scaleCoeff2  scaleCoeff3
    5.25314      0.02087     -0.04029
       Shape Parameters:
shapeCoeff1
     0.1892
  Dependence Parameters:
   range    smooth       DoF
226.7903    0.3562    3.9615
...
```

Model checking

- When you want to check your fitted max-stable model, you usually want to check if
  - observations at each single location are well modelled: return level plot;
  - the dependence is well captured: extremal coefficient function.
- This can be done using a single line of code `plot(M0)`.

Model checking

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  - the dependence is well captured: extremal coefficient function.
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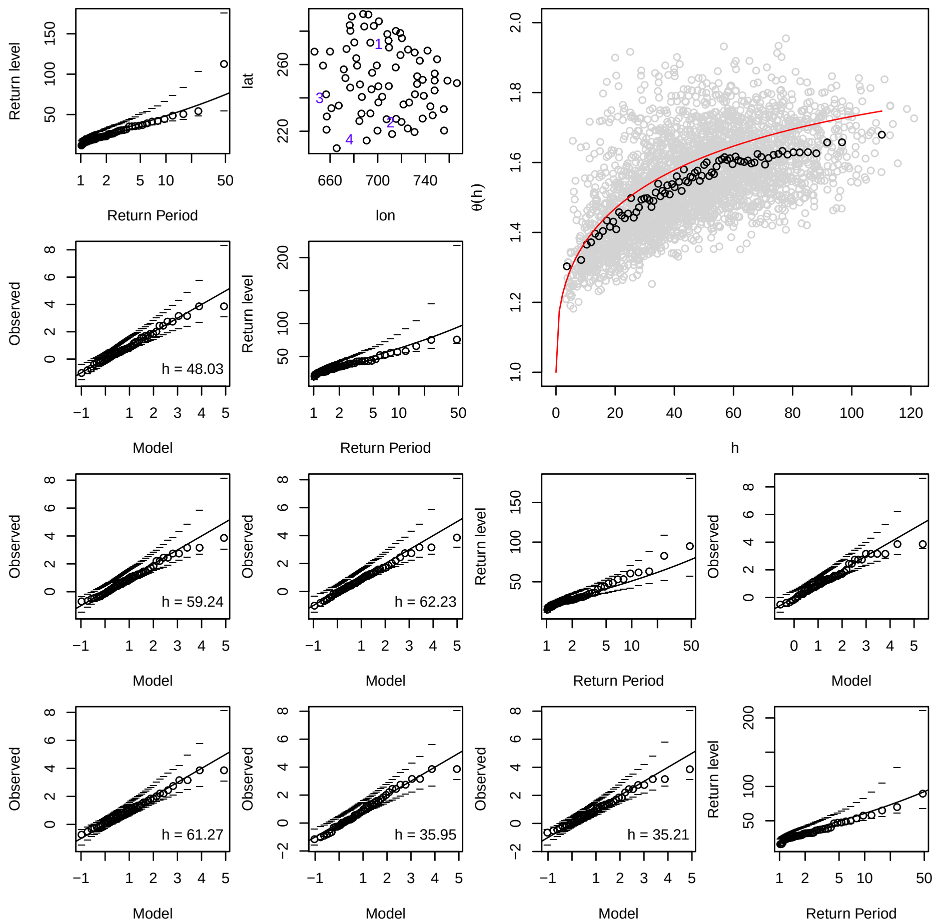

Figure 8: *Model checking for a fitted max-stable process having trend surfaces.*

Predictions

- Prediction works as for the *spatial GEV model* thanks to the `predict` function.
- But beware these predictions are pointwise—no spatial dependence at all!!!
- If you want to do take into account spatial dependence then you need to simulate from your fitted model—see `simulationFinal.R`.

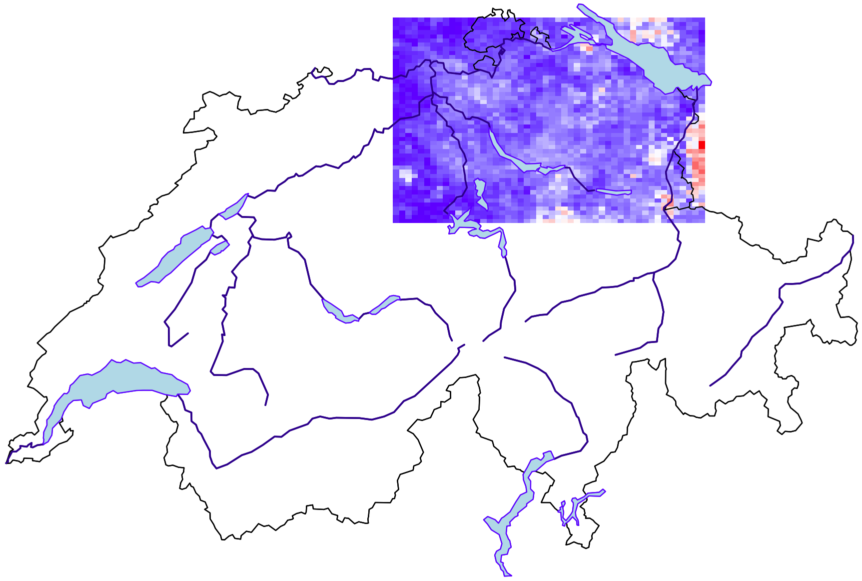

Figure 9: *One simulation from our fitted extremal–t model with trend surfaces.*

1. Data and descriptive analysis
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Fitting
Model checking
Predictions
▷ Homework
5. Conclusion

- Perform a simulation study to estimate the distribution of

$$\sup_{x \in b(\text{Zurich}, 10\text{km})} Z(x),$$

where $b(\text{Zurich}, 10\text{km})$ denotes the ball centred in Zurich with radius 10km;
- Try to redo what was done in *paper1.pdf*;
- Try to redo what was done in *paper2.pdf*;

1. Data and descriptive analysis

2. Simple max-stable processes

3. Trends surfaces

4. General max-stable processes

▷ 5. Conclusion

5. Conclusion

What we haven't seen

- Many (many!) utility functions. Highly recommended to have a look at the documentation;
- The package has a vignette: `vignette("SpatialExtremesGuide")`;
- Copula models—although I do not recommend their use for spatial extremes;
- Bayesian hierarchical models;
- Conditional simulations—really CPU demanding.

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THANK YOU!