Statistical climatology

Some standard statistical tests

t- and z-tests

Let $X_1,\ldots,X_n$ be iid normal random variables with unknown mean μ and unknown variance σ^2. If we are interested in testing the null hypothesis $\mu = \mu_0$ against the alternativ $\mu \neq \mu_0$ we can use the test which rejects if

$$\frac{\left|\frac{1}{n}\sum_{i=1}^{n} X_i - \mu_0\right|}{\sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(X_i-\bar{X})^2}} > t_\alpha(n-1)$$

where $t_\alpha(n-1)$ is the upper a-quantile of the t-distribution with $n - 1$ degrees of freedom. This is called the *t-test*, and was introduced by the English brewery statistician William Gosset (who worked for Guiness brewery in Dublin and wrote under the pen-name Student) in 1908. For large values of n, this can be taken as the corresponding normal quantile. The test is then called a *z-test*.

What if the data are not normal? By the central limit theorem and the law of large numbers, for large values of n the same ratio is approximately normal, so the z-test can be used. If n is small (say ≤20), and the distribution is skewed, this approximation will not be very good. If the distribution is symmetric, the z-test works well, and since the t-quantiles are larger than the normal quantiles, using the t-test as described above is conservative, in the sense that the true level is no larger than α.

F-test

The F-test, named by the American statistician Snedecor after the British statistician Ronald Fisher in 1934, was developed to compare different estimates of variance. Again, we assume that $X_1,\ldots,X_n$ are iid normal random variables with unknown mean μ and unknown variance σ_1^2. If we also have $Y_1,\ldots,Y_m$ iid normal random variables with mean μ and variance σ_2^2 , we can test the null hypothesis $\sigma_1^2 = \sigma_2^2$ against the alternative that the two variances are not the same by rejecting the null hypothesis if

$$\frac{\frac{1}{n-1}\sum_{i=1}^{n}(X_i-\bar{X})^2}{\frac{1}{m-1}\sum_{i=1}^{n}(Y_i-\bar{Y})^2} > F_\alpha(n-1,m-1)$$

where $F_\alpha(n-1,m-1)$ is the upper a-quantile of the F-distribution with $n - 1$ and $m - 1$ degrees of freedom.

This test is not valid if the normal distribution fails.