Anomalies, Observed and Simulated Climates

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Some key questions within climatology:

1. Estimation of the uncertainty in climate models
2. Validation of climate models

Main Objectives of The Present Analysis

1. To demonstrate the impact of using anomalies, a departure from a reference value or long-term average, on the variability in an ensemble of models by using reference periods of different lengths.
2. To propose a method to validate a climate model against observed data and to investigate the effect of using different reference periods in observed and climate model data on conclusions about model's validity.

Data analyzed:

1. A sequence of global annual mean temperatures covering the period 1880-2013. The reference period applied is 1951-1980.
2. En ensemble of 26 climate models' simulations of temperature (the RCP 2.6 scenario with lower radiative forcing and reduced greenhouse gas emissions and indirectly emissions of air pollutants). All models were also merged into a single data set with 3,484 observations. 30-yr long reference periods with various initial years were applied.

Theoretical background

Let $X_1, \ldots, X_n$ be dependent random variables, such that $X_i \sim F_i$ with some distribution function F_i. Consider

$$F_n(x) = \frac{1}{n}\sum \mathbb{1}\{X_i \le x\} \qquad (1)$$

an empirical distribution function, where $\mathbb{1}(A)$ is the indicator of event A. Then the following holds [2]:

1. $\mathbb{E}(F_n(x)) = \bar{F}_n(x) := \frac{1}{n}\sum_{i=1}^{n} F_i(x)$,
2. $\mathbb{V}ar(F_n(x)) = \frac{1}{n^2}\sum\sum \mathbb{C}ov\big(\mathbb{1}(X_i \le x), \mathbb{1}(X_j \le x)\big)$,
3. $\mathbb{P}\big(|F_n(x) - \hat{F}_n(x)| > \epsilon\big) \to 0$ as $n \to \infty$ for any $\epsilon > 0$ provided $\mathbb{V}ar(F_n(x)) \to 0$.

In addition let $Y_1, \ldots, Y_m$ be dependent random variables, such that $Y_i \sim G_i$ and $\mathbb{C}ov(X_i, Y_i) = 0$. Assume that distribution F_i differs from G_i only by a shift of location, i.e. $F_i(x) = G_i(x + \Delta)$. The shift parameter Δ may be estimated [1]:

$$\widehat{\Delta}(x) = G_m^{-1}(F_n(x)) - x. \qquad (2)$$

By studing a simultaneous $(1-\alpha) \times 100\%$ confidence band for $\widehat{\Delta}(x)$ (see [3] page 146 for the description of the construction method), one may conclude (1) whether the shift model is appropriate or not (by checking if a horizontal line might be drawn within the band), (2) whether the difference is significant or not (by checking if the band covers zero), and (3) at which quantile(-s) and by how much the distributions differ.

The Kolmogorov-Smirnov test statistic, $D_{n,m}$, is defined as follows:

$$D_{m,n} = \sup_x |F_n(x) - G_m(x)|. \qquad (3)$$

Results. Objective 1

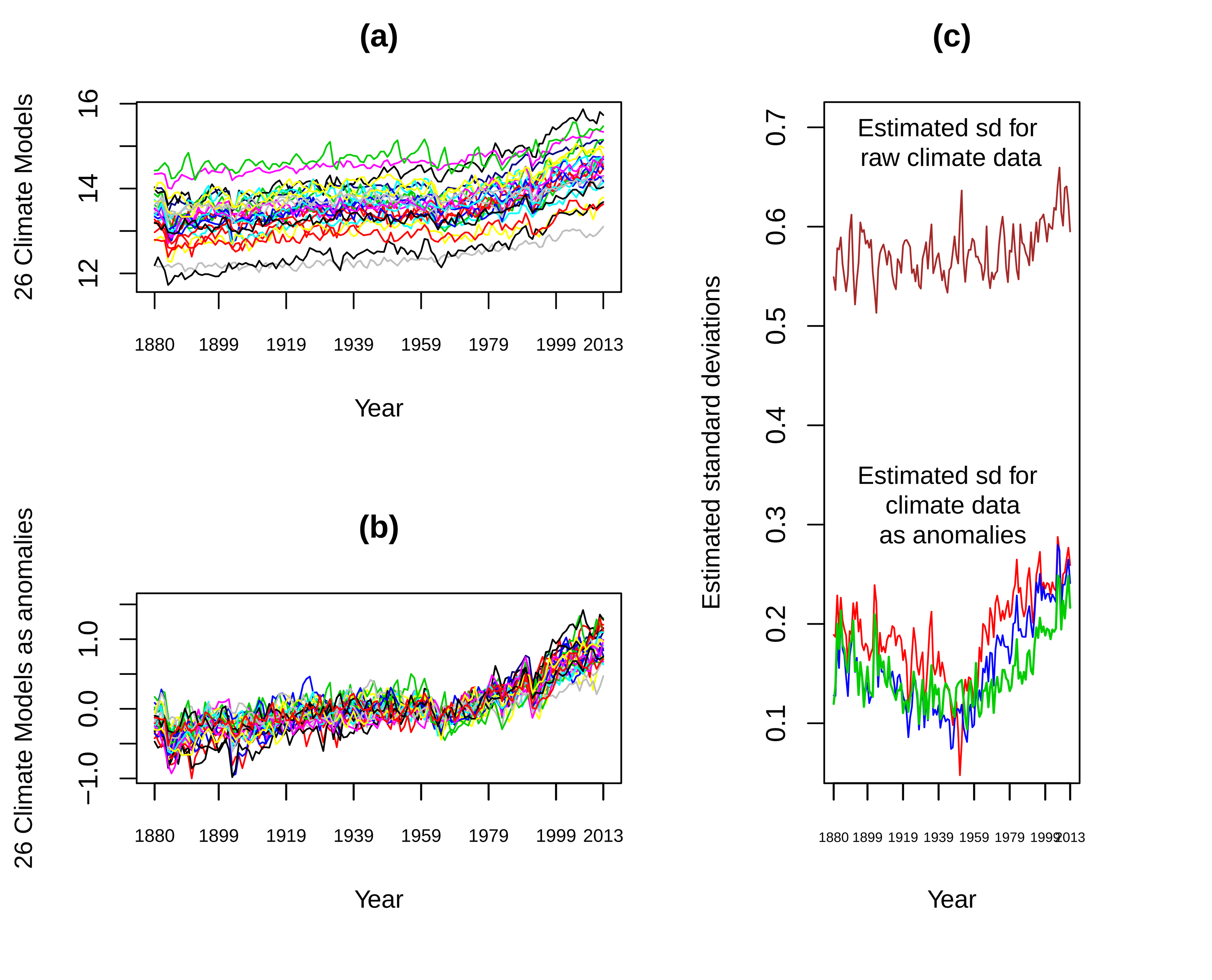

Figure 1: *(a): 26 climate model simulations as raw observations, (b): the same simulations in form of anomalies with the reference period 1937-1965, and (c)*: estimated standard deviation for raw simulations and for anomalies with reference periods 1950-1952 (in red), 1937-1965 (in blue) and 1896-2006 (in green).

The substantial decrease in the variability in the ensemble is obvious. Further, the shorter the period, the smaller the variance during this period.

Objective 2

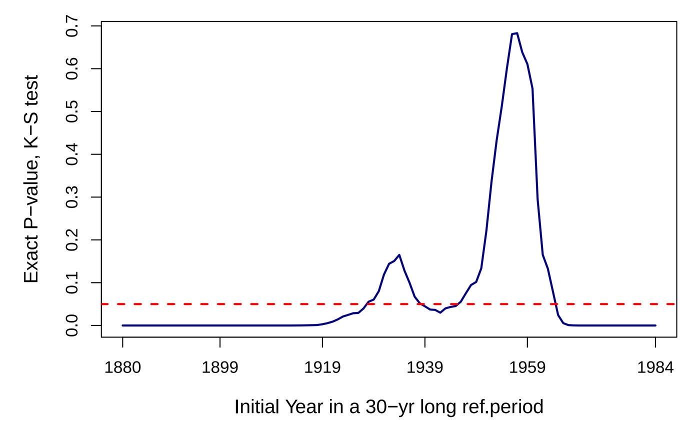

Figure 2: Exact p-values for the K-S test for the entire time series when climate data has variuos 30-yr long reference periods.

Both Figure 2 and Figure 3 points out that the difference is statistically insignificant when the reference period is the same, but highly statistically significant in the case of extremely different reference periods.

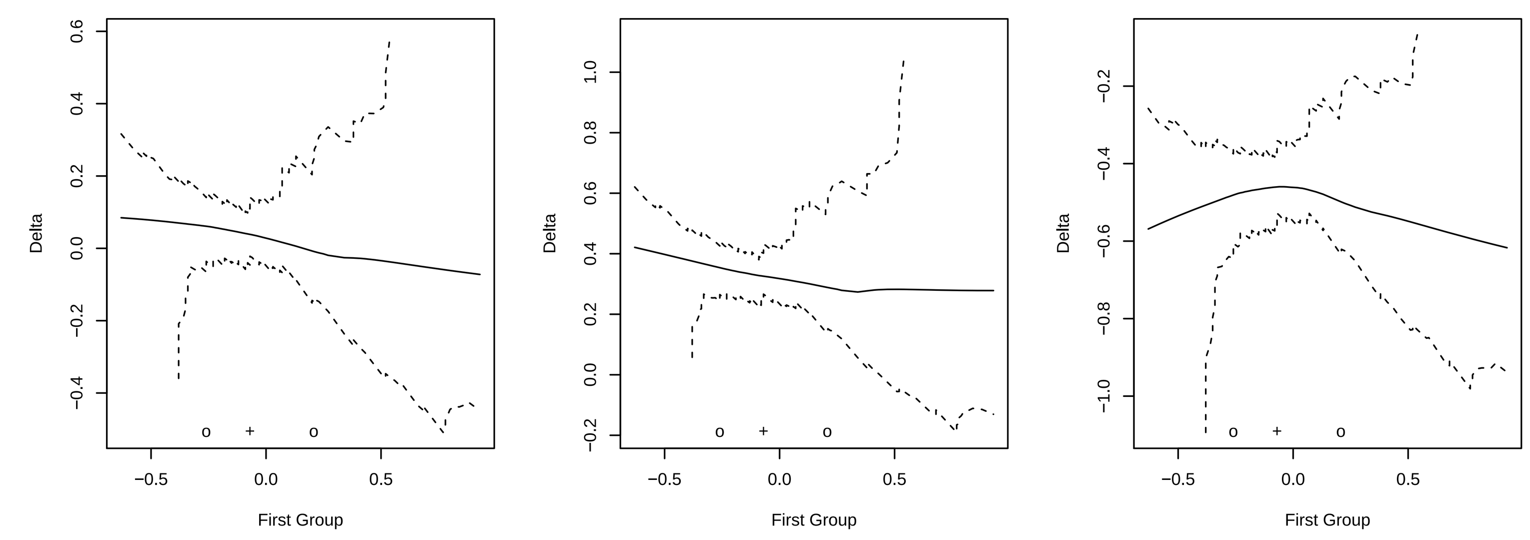

Figure 3: *Three estimated Δ-functions with associated 95% confidence bands for* `Merged` *data set where the ref. periods are 1951-1980 8to the left), 1880-1909 (in the center) and 1984-2013 (to the right).* The $+$ along the x-axis marks the position of the median in the first group, that is the sample of observed temperature, and the lower and upper quartilies are indicated by an $\circ$.

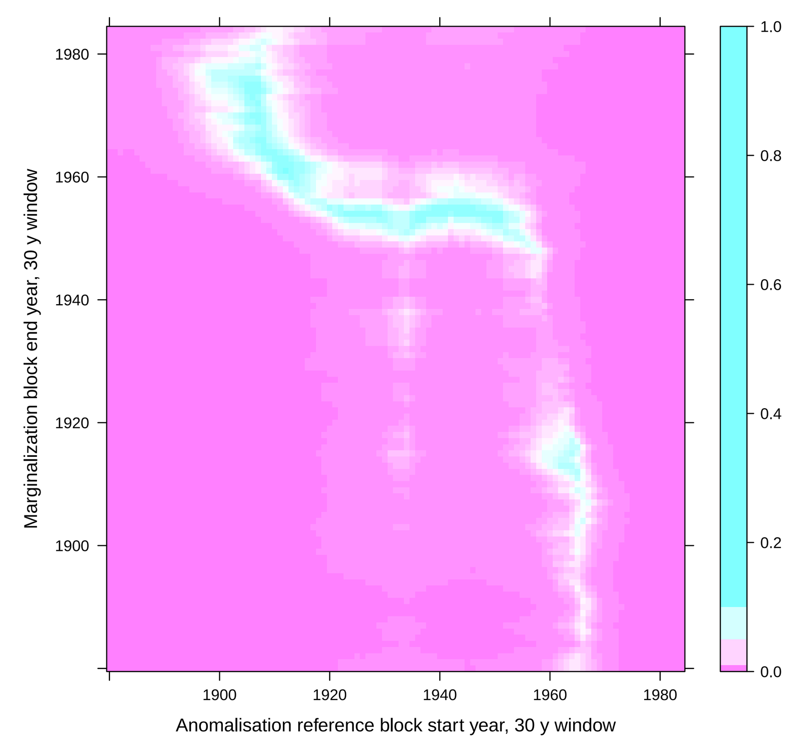

Figure 4: P-values for the K-S test for the various 30-yr long time blocks when climate data has variuos 30-yr long reference periods

The block design makes it possible to compare distributions over different time periods and by that provides an additional information about equality of distributions over these periods.

Conclusions

- The anomalies do decrease the variabilty in an ensemble of climate models regardless of how long the reference period is.
- If the reference period for model data is far away from the reference period for observed data, it is very likely to draw erroneous conclusions about the difference between two types of data.

Possible Topics of Future Research:

1. In the case raw data are not available, to propose method(-s) of adjusting samples with different reference periods, and an analysis of the sensitivity of method(-s) to the time distance between the reference periods.

References

[1] Doksum K.: Empirical probability plots and statistical inference for nonlinear models in the two-sample case, *The Annals of Statistics*, 1974, Vol. 2, No. 2, 267-277.

[2] Shorack G.R.: Convergence of Reduced Empirical and Quantile Processes with Application to Functions of Order Statistics in the Non-I.I.D. Case, *The Annals of Statistics*, 1973, Vol. 1, No. 1, 146-152.

[3] Wilcox R.: Introduction to robust estimation and hypothesis testing, Chapter 5.. DOI: 10.1016/B978-0-12-386983-8.00005-6