INTERNATIONAL JOURNAL OF CLIMATOLOGY
Int. J. Climatol. **33**: 1264–1269 (2013)
Published online 4 May 2012 in Wiley Online Library
(wileyonlinelibrary.com) DOI: 10.1002/joc.3510

Royal Meteorological Society

Estimating daily mean temperature from synoptic climate observations

Yuting Ma[a] and Peter Guttorp[a,b]*
[a] *Department of Statistics, University of Washington, Seattle, WA, USA*
[b] *Norwegian Computing Center, Oslo, Norway*

ABSTRACT: We compare some different approaches to estimating daily mean temperature (DMT). In many countries, the routine approach is to calculate the average of the directly measured minimum and maximum daily temperature. In some, the maximum and minimum are obtained from hourly measurements. In other countries, temperature readings at specific times throughout the day are taken into account. For example, the Swedish approach uses a linear combination of five temperature readings, including the minimum and the maximum, with coefficients that depend on longitude and month. We first look at data with very high temporal resolution, and compare some different approaches to estimating DMT. Then, we compare the Swedish formula to various averages of the daily minimum and maximum, finding the latter method being substantially less precise. We finally compare the Swedish formula to hourly averages, and find that a recalibrated linear combination improves estimation accuracy. Copyright © 2012 Royal Meteorological Society

KEY WORDS bias; Ekholm–Modén formula; linear combination; variability

Received 31 October 2011; Revised 16 March 2012; Accepted 8 April 2012

1. Introduction

There are different ways to calculate daily mean temperature (DMT) at a station from data collected at different times of the day. In many countries, the approach is to average the minimum and maximum temperature observed, although this may be the minimum and maximum hourly readings or the actual minimum and maximum obtained from minimum and maximum thermometers or other devices (see WMO, 2008, for details about temperature measurements). In other countries, a linear combination of measurements taken at different times of the day is used, sometimes including the minimum and maximum as well. For example, the Scandinavian countries each have a different linear combination of data, depending on the frequency of recorded observations (Nordli *et al.*, 1996, Appendix II). When using temperature data for climatological purposes, such as calculating the uncertainty of estimates of global mean temperature, it is important to take into account the variability of the method used to calculate DMT. The purpose of this paper is to present a case study quantifying the difference between some DMT estimators.

There has not been much work on comparing the different approaches to estimating DMT. Hovmöller (1960) discusses how to adjust historical Icelandic data so as to be useful for climatological purposes, based on a variety of different observational schedules.

Weiss and Hays (2005) compare hourly average (taken as ground truth), 3-hourly average, average of min and max, a weighted average, and a method used in the CERES crop simulation program that uses a cubic interpolation between min and max. The goal of their paper was to see the effect of different DMT computations when used as input to a highly nonlinear algorithm. The 3 h average performed best in their context. However, few synoptic networks report that frequently.

Reicosky *et al.* (1989) looked at five different ways of computing the diurnal hourly temperature curve based on observing only the minimum and the maximum. They found that such methods worked better on clear than on cloudy days.

Here, we will look at different ways of combining synoptic temperature measurements to estimate the DMT. We will mainly focus on Swedish measurements at a few stations in the SMHI synoptic network (http://www.smhi.se/klimatdata/meteorologi/dataserier-for-observationsstationer-1961-2008-1.7375). Figure 1 shows the locations of the stations.

The standard Swedish approach dates back at least to 1914 (Ekholm, 1914), and in its current form has been in use since 1947 (Nordli *et al.*, 1996). It is called the Ekholm–Modén (EM) formula, and is a linear combination of the daily minimum, the daily maximum, and measurements at 6, 12, and 18 h UTC. The maximum and minimum both correspond to the time period 18 h UTC the previous day until 18 h UTC the current day. Swedish time is UTC+1 in the winter, UTC+2

*Correspondence to: P. Guttorp, University of Washington, Box 354322, Seattle, WA 98195-4322, USA.
E-mail: peter@stat.washington.edu

Figure 1. Observation stations used.

in the summer (last Sunday of March through last Sunday of October). We then can write the formula, using observations at Swedish standard time, as $T_{\mathrm{mean}} = aT_{07} + bT_{13} + cT_{19} + dT_{\max} + eT_{\min}$. The coefficients of the linear combination depend on month and longitude, although the longitude dependence is relatively small, and can be found in Alexanderson (2002) or online at http://www.smhi.se/kunskapsbanken/meteorologi/koefficienterna-i-ekholm-modens-formel-1.18371. They were essentially derived by least squares (LS) fitting to stations with hourly data available (Ekholm, 1914; Modén, 1939). It is interesting that the coefficients are restricted to sum to one. Apparently this originates in numerical work in the early part of the 20th century, where this constraint stabilized the LS calculations (Ekholm, 1914). Also, the coefficient d for the maximum daily temperature is always set to 0.1, regardless of month and longitude. We have not found any reason for this constraint in the literature. In Ekholm's and Modén's original papers, the maximum temperature was not included (i.e. $d = 0$).

We begin in Section 2 by looking at the accuracy and precision of various estimates of DMT compared to an estimate from a high resolution (1 min) data set. We do not have access to 1 min data at any of the Swedish synoptic stations, although hourly data are available at some of them. In Section 3, we compare the EM formula to various linear combinations of the minimum and maximum, and note that the latter generally are substantially more variable. Section 4 is devoted to comparing the EM formula to hourly averages for two stations. We discuss our findings in the final Section 5, and describe some future research.

2. Hourly measurements compared to high frequency measurements

The highest temporal resolution measurements we have access to have 1 min resolution. The question of interest in this section is how accurate the average of hourly measurements is compared to the average of the 1 min data. We look at data from the air traffic control tower at Visby airport (57.673N, 18.345E, altitude 49 m). Figure 2 shows the average daily temperature curve (minute-by-minute) for this station, averaged over the days in the months of January and June 2010.

It is clear from Figure 1 that the daily temperature curves are not symmetric about the average daily temperature, and that a sine curve is not a particularly good fit, particularly in January.

By assuming the daily average of 1 min temperatures as the true value of DMT, we compare different approaches to calculate daily temperatures and study their bias and variability. The first method we consider is to take the average of hourly temperatures (every 60th observation in 1 min data), which should have relatively small bias due to its high frequency of measurements. The second one is averaging daily minima and maxima, the method that is broadly used. And we also use the original monthly EM formula given by SMHI. At last, we apply a linear combination of EM type to 1 min data and get coefficients by using an LS fit to both sets of data in January and July. The resulting coefficients are used to generate estimated DMT, which are used to compare with the previous three methods. The comparison results are summarized in Table I.

As shown in the Table I, DMT estimated by the hourly average results in the smallest bias as compared to daily minute average temperatures. Using average of daily minima and maxima has the largest bias and variability among the four estimators, implying less adequacy and stability in estimating DMT. The DMT estimated by the original EM formula also shows relatively large discrepancy from its true value, especially during summer time. Finally, DMT estimated by LS coefficients based on the EM formula has a small bias that is very close to that of hourly averages, though with slightly greater variability. It is also noticed that the LS coefficients method has better performance in January than in July, which mostly is due to a greater variation in daily temperatures during summer time.

3. Comparison of EM to linear combinations of maximum and minimum

In this section, we will take T_{mean} as the true value, and look at what linear combinations of $T_{\min}$ and $T_{\max}$ provide

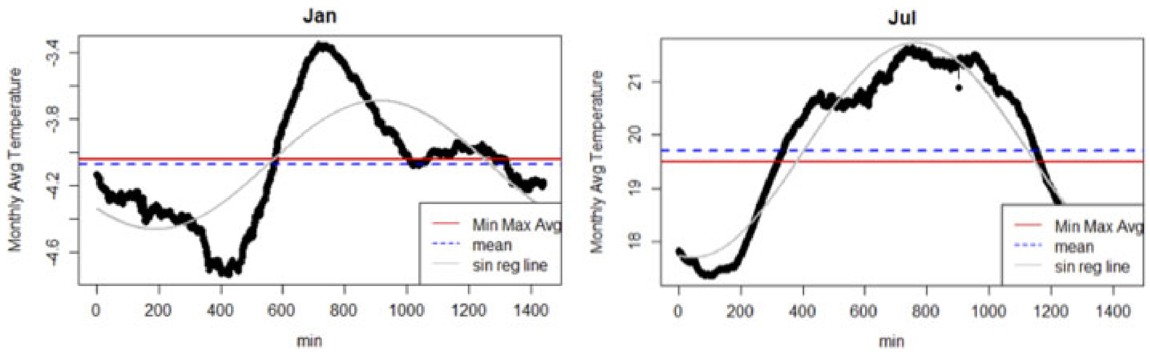

Figure 2. Daily average temperature curves for Visby air traffic control tower for January (left) and July (right) using minute data from 2010. Also shown are fitted sine curves, the average daily temperature (dashed horizontal line) and the average of daily minimum and maximum average temperature (solid horizontal line).

Table I. Estimator comparison results (bias, 95% CI and SD) for Visby station by using 1 min data during January and July, 2010. We also show p-value for the hypothesis of equal means.

Estimator	January					July				
	Bias	95% CI		SD	p-Value	Bias	95% CI		SD	p-Value
LS coefficients formula	−0.002	−0.084	0.08	0.22	0.96	0.05	−0.19	0.29	0.66	0.68
EM formula	−0.018	−0.22	0.18	0.55	0.86	−0.54	−0.97	−0.10	1.19	0.02
Average of daily minima and maxima	0.09	−0.06	0.25	0.42	0.23	0.06	−0.44	0.56	1.36	0.81
Hourly average	−0.001	−0.022	0.021	0.059	0.93	0.001	−0.024	0.026	0.069	0.94

CI, confidence interval; SD, standard deviation.

the best approximation to T_{mean} in the LS sense based on 49 years of observations on Stockholm–Bromma (59.35N,17.95E) and Sundsvall (62.52N, 17.44E) airports. We consider four different models. The first compares T_{mean} to T_{ave}, the average of $T_{\min}$ and $T_{\max}$. In the second, we find the coefficient f that minimizes the sum of squared differences between T_{mean} and $T_{\text{ave}}^{(1)} = fT_{\min} + (1-f)T_{\max}$. In the third, we find the coefficients g and h that minimizes the sum of squared differences between T_{mean} and $T_{\text{ave}}^{(2)} = gT_{\min} + hT_{\max}$. Finally, the fourth estimator is the same as the third, but allowing the coefficients to change with the month. To examine the resulting linear combinations of $T_{\min}$ and $T_{\max}$, we apply them to the data of 2009 (which were not used in the fitting) and compare the PRMSE from the T_{mean} given by SMHI. Table II contains the results.

From the table, we see no salient differences among the predictions of T_{ave} in 2009 of the first three linear combinations of $T_{\min}$ and $T_{\max}$. Notice that, although T_{ave} is a special case of $T_{\text{ave}}^{(1)}$ which in turn is a special case of $T_{\text{ave}}^{(2)}$, the PRMSE for 2009 are not necessarily monotone, since the coefficients are based on earlier data. The PRMSEs from T_{mean} are all large (of the order of 1 °C) and different annual linear combinations do not show substantial improvement over the average of $T_{\min}$ and $T_{\max}$. Using monthly coefficients rendered, for both Stockholm and Sundsvall, somewhat larger decreases of PRMSE, of course at the cost of estimating more parameters. A comparison between the annual and monthly coefficients for the two stations is shown in Figure 3. The monthly coefficient g for $T_{\max}$ reaches its highest value and the coefficient h for $T_{\min}$ reaches its nadir around July (summer time) and the reverse occurs around February (winter time).

4. The EM formula compared to average of hourly measurements

The EM formula was developed using a few stations with hourly measurements with mean of the hourly observations as ground truth (Ekholm, 1914; Modén, 1939). Hence, it appears sensible to compare the formula to the daily average for some current hourly stations, not used in the original determinations. We do this for Malmö (55.57N, 13.07E) and Stockholm Observatory (59.34N, 18.06E). In addition, we do a recalibration of the formula for these stations.

Using the EM formula described in Section 1, we use LS optimization (function *nls* in R; R Development Team, 2011) to derive the best linear combination of T_{07}, T_{13}, T_{19}, $T_{\min}$, and $T_{\max}$ by taking T_{mean} as the true value. To simplify the calculations, we divide the data into 3 month seasons (rather than months) with winter being December–January–February, etc. The seasonal LS coefficients for Stockholm and Malmö are listed in Table III.

To examine the existing EM formula, we first compare the given T_{mean} from the SMHI synoptic network and the observed daily hourly average temperature, which we set as the truth for DMT for both Malmö and Stockholm. We

Table II. Comparison of different linear combinations of T_{min} and T_{max} to approximate T_{mean}.

Method	Stockholm				Sundsvall			
	Estimates		Bias	PRMSE	Estimates		Bias	PRMSE
$T_{ave} = (T_{min} + T_{max})/2$	–		0.05	0.810	–		0.035	1.056
$T_{ave}^{(1)} = fT_{min} + (1 - f)T_{max}$	$f = 0.503$		0.028	0.804	$f = 0.495$		0.077	1.064
$T_{ave}^{(2)} = gT_{min} + hT_{max}$	$g = 0.505$		0.025	0.798	$g = 0.494$		0.082	1.066
	$h = 0.492$		–	–	$h = 0.491$		–	–
Monthly coefficients of $T_{ave}^{(2)} = gT_{min} + hT_{max}$	0.439	0.567	0.019	0.74	0.436	0.581	0.022	1.022
	0.475	0.523	–	–	0.436	0.585	–	–
	0.472	0.487	–	–	0.454	0.563	–	–
	0.406	0.508	–	–	0.418	0.525	–	–
	0.331	0.563	–	–	0.348	0.548	–	–
	0.322	0.595	–	–	0.297	0.595	–	–
	0.322	0.599	–	–	0.297	0.604	–	–
	0.358	0.577	–	–	0.360	0.566	–	–
	0.418	0.538	–	–	0.443	0.505	–	–
	0.449	0.522	–	–	0.462	0.509	–	–
	0.434	0.536	–	–	0.436	0.594	–	–
	0.443	0.548	–	–	0.443	0.589	–	–

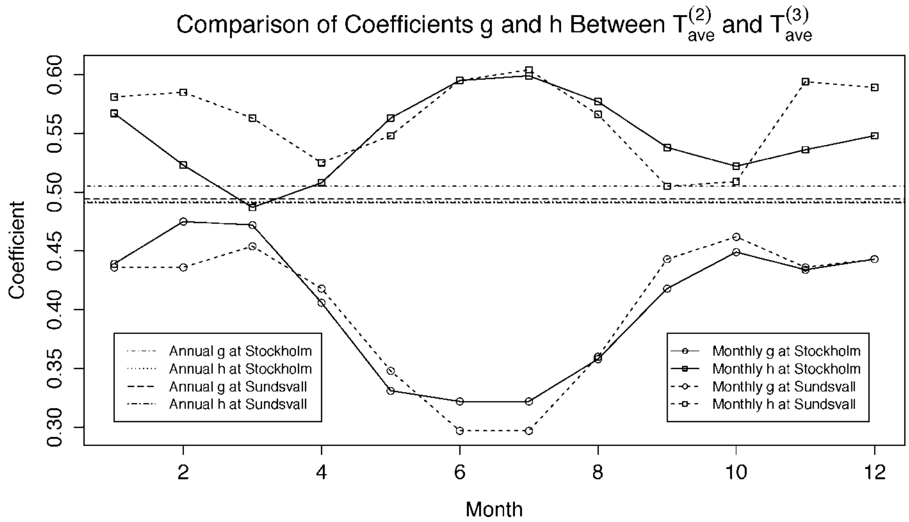

Figure 3. Comparison of coefficients g and h between monthly and annual fits for Stockholm–Bromma and Sundsvall.

find (Table IV) that T_{mean} from both stations have a bias of about 0.1 °C, and is usually smaller than the hourly average temperature.

We would also like to compare the average of T_{min} and T_{max} to hourly average temperatures. In some data sets, such as WMO's Global Surface Summary Of Day (GSOD; http://gosic.org/ios/MATRICES/ECV/ATMOSPHERIC/SURFACE/ECV-GCOS-ATM-SURFACE-airpressure-GSOD-data-context.htm), the maximum and minimum temperatures are calculated from hourly data. Our results (Table IV) show that, as expected, the hourly min and max temperatures are less extreme than the continuously measured T_{min} and T_{max} from SMHI. The absolute differences between the hourly and the continuous values for T_{min} and T_{max} for Malmö are approximately the same, around 0.2 °C; however, for Stockholm there are greater differences between hourly max and T_{max}.

Finally, we investigate the differences between different methods of estimating DMTs, by using the original EM formula, and by using the seasonal LS coefficients we derived above, setting the daily hourly average temperatures as the true values. We also look at the consequences of using the minimum and maximum hourly temperatures instead of the actual minima and maxima. We see that the EM coefficients incur a bias of up to 0.1 °C, and that by using LS coefficients, the bias is halved. The standard errors are substantially decreased, implying better stability of estimation by using the LS coefficients. Therefore, we conclude that our seasonal LS coefficients may provide more accurate estimates of DMTs than the currently used coefficients. The confidence intervals given are computed without taking into account the serial dependence of the data, and are likely somewhat too short.

Table III. Seasonal LS coefficients for Stockholm and Malmö (unitless).

Station	Season	*a*	*b*	*c*	*d*	*e*
Stockholm	Spring	0.22	0.16	0.24	0.16	0.22
	Summer	0.20	0.20	0.23	0.10	0.27
	Autumn	0.29	0.21	0.30	0.09	0.12
	Winter	0.32	0.17	0.31	0.10	0.10
Malmö	Spring	0.19	0.22	0.21	0.14	0.24
	Summer	0.18	0.20	0.27	0.09	0.26
	Autumn	0.30	0.20	0.28	0.11	0.13
	Winter	0.32	0.17	0.32	0.09	0.10

5. Discussion

The practice of averaging minimum and maximum temperature to estimate a DMT assumes that the diurnal cycle is symmetric throughout the year. We have seen that the variability of this estimator is substantially larger than one that in addition uses temperature readings from throughout the day. WMO (2010) recommends use of this estimator in spite of these drawbacks, saying 'Even though this method is not the best statistical approximation, its consistent use satisfies the comparative purpose of normals.' It is a difficulty that in many databases different calculations are used for different countries, and furthermore that the calculations often change over time. The definition of maximum and minimum (whether it is measured in continuous time or from hourly observations) can affect both accuracy and precision of this estimator. The main issue here is that the standard error is substantially larger for the WMO-recommended estimator, and this needs to be taken into account when using data where DMT is calculated in this way.

The standard Swedish formula for estimating DMT could probably be improved by fitting the model to the much larger set of hourly measurements available today. In particular, it would be interesting to see whether anything is gained from the longitude dependence of the coefficients.

The practice of using the minimum and maximum hourly temperatures is defensible when estimating DMT using a formula of the EM type, but can add substantially to the bias incurred when just averaging minimum and maximum temperature. A further problem with the latter method is that different countries have different conventions in what hours the extreme values are computed for. Thus, direct comparisons of these estimates between countries are not so easy. A change in the definition of the climate day can make differences of up to 20 °C (Hopkinson *et al.*, 2011) at a single station. Another difficulty with the max–min values is that when observations are not automated, the precision of the instrument is different from a regular thermometer, and it has to be taken out of the screen and reset every day, which can lead to drift in calibration, and stretches of missing values.

Generally temperature series tend to exhibit so-called long-term memory (Beran, 1994). This implies that standard error that assumes independent observations (or even autoregressive dependence structure) underestimate the true variability. This dependence structure is partly due to decadal modes of variability, and partly to the oceans' heat storing capacity. A rough estimate of the difference in standard errors is a factor of 3, based on the approach by Craigmile *et al.* (2004).

Acknowledgements

This work had partial support from the National Science Foundation DMS-1106862. The authors are grateful for constructive comments from Paul Whitfield, Erik Kjellström, and Trausti Jónsson, and for data from Gunnar Berglund at SMHI.

Table IV. Estimator comparisons for Stockholm and Malmö (units °C).

Station	Comparison	Bias	CI		SD
Stockholm	T_{mean} *versus* hourly average	0.148	0.11	0.18	0.34
	T_{min} *versus* hourly min	−0.055	−0.08	−0.02	0.30
	T_{max} *versus* hourly max	0.171	0.09	0.25	0.79
	DMT with hourly min and max *versus* with real min & max	−0.032	−0.06	0.00	–
	DMT by original coefficients *versus* hourly average[a]	0.1	0.07	0.14	–
	Estimated DMT by LS coefficients *versus* hourly average	0.042	0.00	0.08	–
Malmö	T_{mean} *versus* hourly average	−0.020	−0.04	0.01	0.32
	T_{min} *versus* hourly min	−0.269	−0.30	−0.24	0.26
	T_{max} *versus* hourly max	0.259	0.20	0.32	0.58
	DMT with hourly min and max *versus* with real min and max	0.103	0.08	0.13	–
	Estimated DMT by original coefficients *versus* hourly average	0.06	0.02	0.10	–
	Estimated DMT by LS coefficients *versus* hourly average	0.004	−0.03	0.03	–

CI, 95% confidence interval of the difference between two values in question; max, maximum; min, minimum; SD, standard deviation of the differences between two values in question.
[a] Original monthly coefficients from EM formula are converted to seasonal coefficients by taking the averages of monthly coefficients.

References

Alexanderson H. 2002. Temperatur och nederbörd i Sverige 1860–2001. Sveriges meteorologiska och hydrologiska institut, Meteorologirapport 104.

Beran J. 1994. *Statistics for Long-Memory Processes*, Chapman & Hall/CRC: Boca Raton.

Craigmile PF, Guttorp P, Percival DB. 2004. Trend assessment in a long memory dependence model using the discrete wavelet transform. *Environmetrics* **15**: 313–335.

Ekholm N. 1914. Beräkning av luftens månadsmedeltemperatur vid de svenska meteorologiska stationerna. Bihang till *Meteorologiska iakttagelser i Sverige*. Band 56, 1914. Almqvist & Wiksell: Stockholm, 110.

Hopkinson RF, Mckenney DW, Milewska EJ, Hutchinson MF, Papadopol P, Vincent LA. 2011. Impact of aligning climatological day on gridding daily maximum–minimum. *J Appl Met Clim* **50**: 1654–1665.

Hovmöller E. 1960. Climatological information on Iceland. United Nations report TAO/ICE/4. Available at http://www.vedur.is/media/vedurstofan/utgafa/greinargerdir/1995/Climatological1960.pdf. (Retrieved March 14, 2012).

Modén H. 1939. Beräkning av medeltemperaturen vid svenska stationer. Statens meteorologisk-hydrografiska anstalt. Meddelanden, serien Uppsatser, no. 29.

Nordli PØ, Alexandersson H, Frisch P, Førland E, Heino R, Jónsson T, Steffensen P, Tuomenvirta H, Tveito OE. 1996. The effect of radiation screens on Nordic temperature measurements. DNMI Report 4/96 Klima.

R Development Core Team. 2011. *R: A Language and Environment for Statistical Computing*, R Foundation for Statistical Computing: Vienna, ISBN 3-900051-07-0. Available at http://www.R-project.org/. (Retrieved March 14, 2012).

Reicosky DC, Winkelman LJ, Baker JM, Baker DG. 1989. Accuracy of hourly air temperatures calculated from daily minima and maxima. *Agric For Meteorol* **46**: 193–209.

Weiss A, Hays CJ. 2005. Calculating daily mean air temperatures by different methods: implications from a non-linear algorithm. *Agric For Meteorol* **128**: 57–69.

WMO. 2008. *WMO Guide To Meteorological Instruments And Methods Of Observation*, Part I, Chapter 2, 7th edn, Retrieved August 5, 2011. Geneva: World Meteorological Organization. Available at http://www.wmo.int/pages/prog/www/IMOP/publications/CIMO-Guide/CIMO_Guide-7th_Edition-2008.html.

WMO. 2010. *Guide to Climatological Practices WMO-No. 100*, 3rd edn, Retrieved October 24, 2011. Geneva: World Meteorological Organization. Available at http://www.wmo.int/pages/prog/wcp/ccl/guide/documents/WMO_100_en.pdf.

