The United Nations Probabilistic Population Projections: An Introduction to Demographic Forecasting with Uncertainty

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PREVIEW *The United Nations publishes projections of populations around the world and breaks these down by age and sex. Traditionally, they are produced with standard demographic methods based on assumptions about future fertility rates, survival probabilities, and migration counts. Such projections, however, were not accompanied by formal statements of uncertainty expressed in probabilistic terms. In July 2014 the UN for the first time issued official probabilistic population projections for all countries to 2100. These projections quantify uncertainty associated with future fertility and mortality trends worldwide.*

This review article summarizes the probabilistic population projection methods and presents forecasts for population growth over the rest of this century.

THE UNITED NATIONS PROBABILISTIC POPULATION PROJECTION METHOD

The United Nations makes projections of the populations of all countries by age and sex and publishes these in the biennial publication *World Population Prospects* (United Nations, 2013). These projections are widely used by international organizations, governments, and researchers for planning and decision making. The UN projections of national populations are based on a cohort component projection method, also referred to as the Leslie matrix method (Leslie, 1945; Preston and colleagues, 2001).

The Cohort Component Projection Method

In the UN cohort component projection method, a population is projected from its base year (set in the UN projections at mid-2010) out to mid-2100 by five-year periods. The main inputs to the population projections are

- future fertility rates,
- survival probabilities, and
- migration counts

All are provided by five-year age groups and gender.

Given those inputs, the cohort component method carries out basic demographic accounting. For example, the population by age and sex in year t+5 is equal to the population in year t plus the intervening births and net migration, minus the intervening deaths.

Traditionally, the UN produced deterministic population projections and issued point projections. These point projections were supplemented with ranges based on different scenarios of demographic changes. In July 2014 the UN for the first time issued official probabilistic population projections, which quantify the uncertainty associated with the demographic projections.

Probabilistic Projections

In the probabilistic projection method, uncertainty in future demographic outcomes is assessed by constructing a large sample of future trajectories for outcomes such as total population size. Then, for each year in the future, point projections are given by the median outcome of the sample of trajectories, while other percentiles of the sample are used to construct prediction intervals.

As an illustration, **Figure 1** shows the probabilistic projections for total population in Burkina Faso. The solid red line displays the point projections based on the median trajectory. We can see that population is projected to increase from 15 million in 2010 to 75 million in 2100.

Key Points

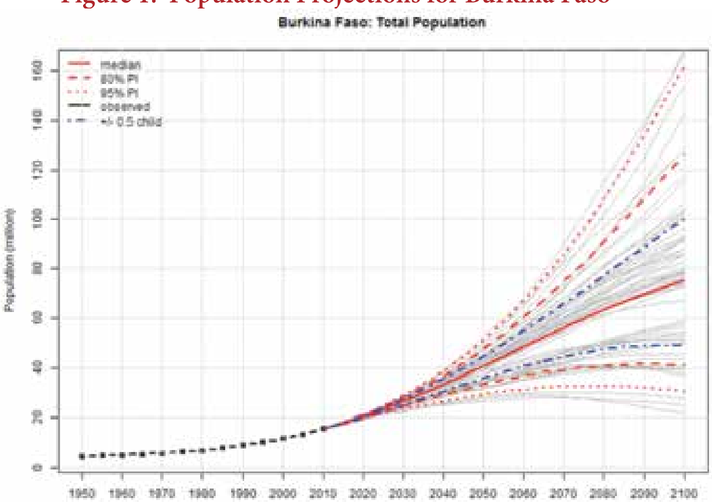
- The UN has now produced probabilistic population projections for countries and regions worldwide which quantify the uncertainty associated with future demographic outcomes such as population growth and age dependency ratios.

- The population projections are constructed with a cohort component projection method, which projects populations by sex and five-year age groups based on hypothetical future trajectories of fertility, mortality, and migration. Uncertainty surrounds these trajectories, however.

- In the probabilistic projection method, the uncertainty of future demographic outcomes is quantified by constructing a large sample of future trajectories for fertility and mortality outcomes for each country. For each year in the future, point projections are given by the median outcome of the sample of trajectories. The percentiles of the sample are used to construct prediction intervals.

- Probabilistic projections of country-specific fertility and mortality rates are constructed using Bayesian hierarchical models. In such models, estimates of country-specific parameters, such as the pace of a future fertility decline, are based on observed trends in the country as well as on those in other countries.

Figure 1. Population Projections for Burkina Faso



However, there is considerable uncertainty about this projection: shown in gray is a subset of the sample of trajectories. The dashed red lines bound the 80% prediction interval for the population size in 2100 – which ranges from 41 to 126 million. The 95% percent prediction interval, shown by the dotted red line, is much wider still.

The graph shows clearly that there is considerable uncertainty about this country's future population size. The 80% prediction intervals are also much wider than the projection interval that results from adding and subtracting half a child from the median fertility projection (shown in blue). This scenario-based interval was traditionally included with the UN projections, and referred to as the low and high variants of population projections to illustrate the effect of deviations from the point forecast for future fertility rates. The uncertainty in the future population size exceeds the outcomes associated with the low and high variants in countries where women have three or more children on average and is too wide in countries with lower fertility.

As noted, probabilistic population projections for each country were constructed from a set of trajectories of future outcomes of the main inputs to the cohort component model. The trajectories of the total fertility rate as well as of life expectancy at birth are generated using Bayesian hierarchical models (Alkema and colleagues, 2011; Raftery and colleagues, 2013), which will be explained for the fertility projections in the next section. Then, with one central projection for migration, the cohort component projection method is applied using each set of future fertility and mortality outcomes to produce a set of trajectories of future population outcomes by age and sex.

The forecasting performance of the probabilistic projection method was validated by an out-of-sample test in which data from 1950–1990 were used to predict 1990–2010. In that exercise, the method provided reasonably accurate and well-calibrated probabilistic projections for the 1990–2010 period (Raftery and colleagues, 2012).

Regional Projections

The country-specific probabilistic population projections quantify the uncertainty associated with future trends in fertility and mortality for each country. To construct projections of aggregate outcomes, e.g., for the world population, additional between-country correlations for the total fertility rate are incorporated (Fosdick & Raftery, 2014). Probabilistic projections of aggregate outcomes are particularly useful to assess plausible outcomes of future population growth in major areas and globally. For example, the recent UN projections indicate that stabilization of the world population is unlikely to occur this century (Gerland and colleagues, 2014).

BAYESIAN HIERARCHICAL MODELS

The starting point for obtaining probabilistic projections for age-specific fertility rates are projections of the overall level of fertility for a country- period, summarized by the total fertility rate (TFR).

Total Fertility Rate

The TFR for a specific period is the average number of children a woman would bear in her life if exposed to the age-specific fertility rates prevalent during that period.

In the TFR projection model, the typical evolution of a country's fertility over time is described in three phases, shown in **Figure 2** (Alkema and colleagues, 2011).

Phase I is characterized by high fertility that is stable or increasing. Since all countries have completed Phase I, this phase is not of interest for projections. During Phase II, a shift occurs from high to low fertility, referred to as the fertility transition, which is ongoing in many countries. The post-fertility transition period is referred to as Phase III. Most developed countries are currently in Phase III.

For constructing TFR projections, models were developed to project changes in the TFR during phases II and III, as well as the transition from Phase II to III. In this review, we summarize the models for changes in the TFR in phases II and III.

Decline Functions

Decline functions are used to model the fertility transition during Phase II – the name

Figure 2. Illustration of the Three Phases of the Model for Total Fertility

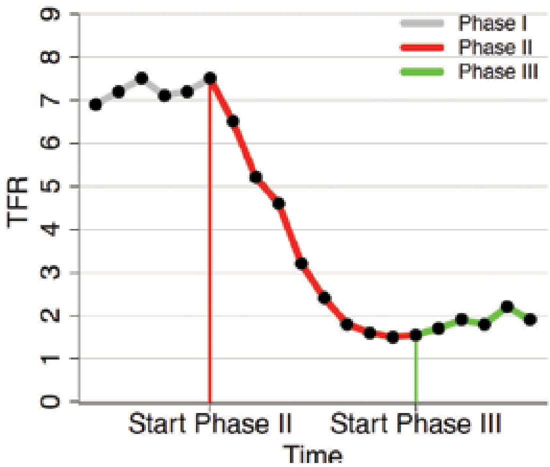
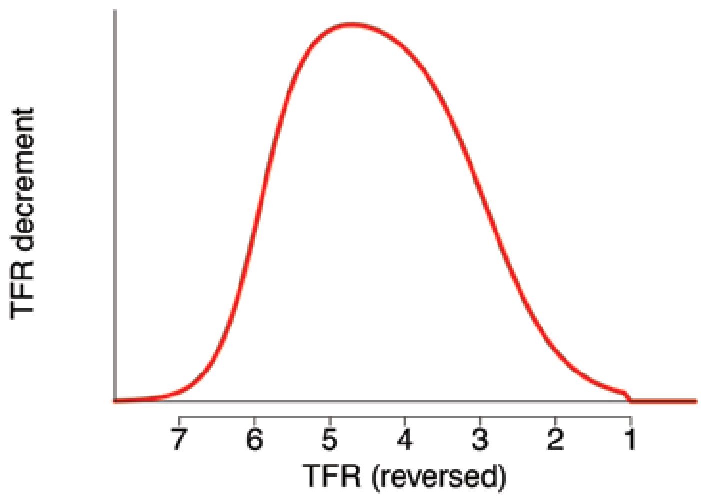


Figure 3. Illustration of Decline Function Used to Project Expected Declines in the TFR during the Fertility Transition (Phase II)



reflecting that a decline function provides the expected decrease in the TFR. An illustrative decline function is shown in **Figure 3**.

The main difficulty in constructing projections of the TFR for countries at the start of their fertility transition lies in the estimation of the five parameters of this function: what set of parameter values would best describe future TFR declines? In the UN probabilistic projection approach, the aim is to obtain parameter estimates that are based both on observed past declines in the country of interest and also on observed declines

The recent UN projections indicate that stabilization of the world population is unlikely to occur this century.

in other countries that have gone through at least part of their fertility transitions.

This aim is accomplished through the use of a Bayesian hierarchical model, the structure of which is described in the Appendix. Bayesian models take prior beliefs about unknown quantities, such as expected changes in the TFR, and revise these based on new information about fertility declines. In the hierarchical model, information is exchanged across countries, based on the assumption that the quantities are drawn from a common worldwide probability distribution.

Application of the hierarchical models results in a set of country-specific estimates of the model parameters and corresponding decline functions, which provide the expected decline at any level of the TFR. The set of country-specific curves is used to construct a set of trajectories of future TFR outcomes. In each trajectory, during Phase II, the projected changes in the TFR are given by the projected country-specific expected declines, combined with distortion terms that capture the difference between true declines and the expected declines.

Once a country has entered the post-transition low-fertility Phase III, the TFR is projected to converge toward and oscillate around country-specific asymptotes. This is modeled using a standard time-series model, a stationary autoregressive process of order one, with country-specific parameters (Raftery and colleagues, 2013). The estimation of the country-specific parameters for the Phase III model was also carried out using a Bayesian hierarchical model, following the same rationale as for the Phase II model: parameter estimates are based on data from the country of interest (for those countries that have reached Phase III before the start of the projection) as well as available Phase III information from other countries.

Decline functions and fertility projections are illustrated in **Figures 4a** and **4b** for Burkina Faso.

The TFR in Burkina Faso decreased from around 7 children per woman in the 1980s to 6.1 in the period 2005-2010. The range of plausible decline functions, which were obtained with the Bayesian hierarchical model, suggest that the expected TFR decrements are likely to stay around 0.4 children per five-year period until the TFR has decreased to around 4, after which the expected decrements are projected to decrease in the final phase of the fertility transition. The TFR is projected to decrease to 2.1 by 2095-2100, with the 80% prediction interval given by (1.5, 3.0).

PROJECTIONS FOR MAJOR AREAS

This review summarized the probabilistic population projection method that is currently used by the UN for producing global population projections with uncertainty assessments. The addition of an uncertainty assessment to the UN population

Figure 4a. Decline Functions for Burkina Faso

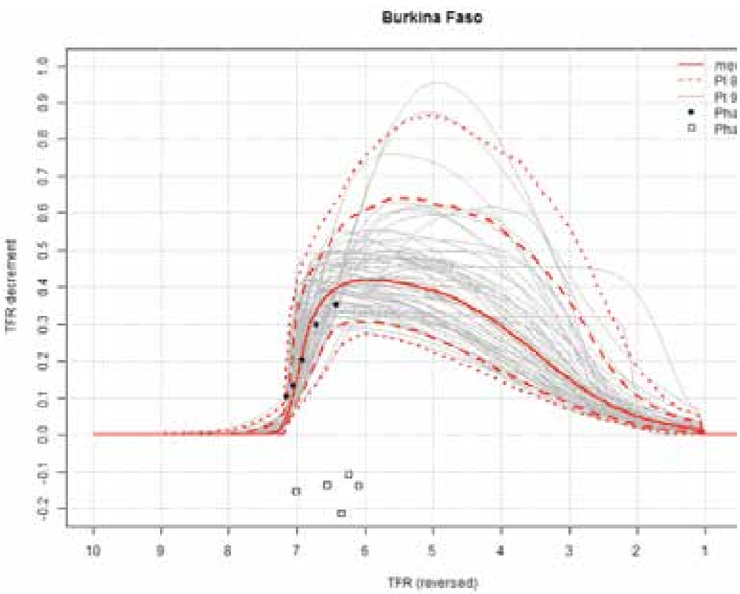
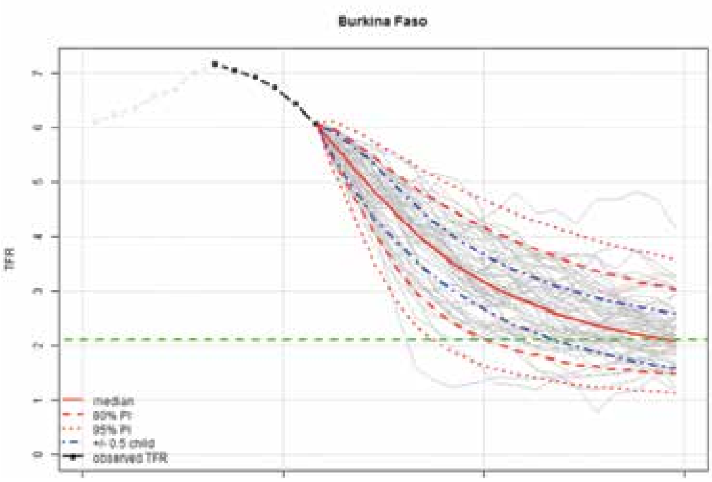


Figure 4b. The TFR Projections for Burkina Faso

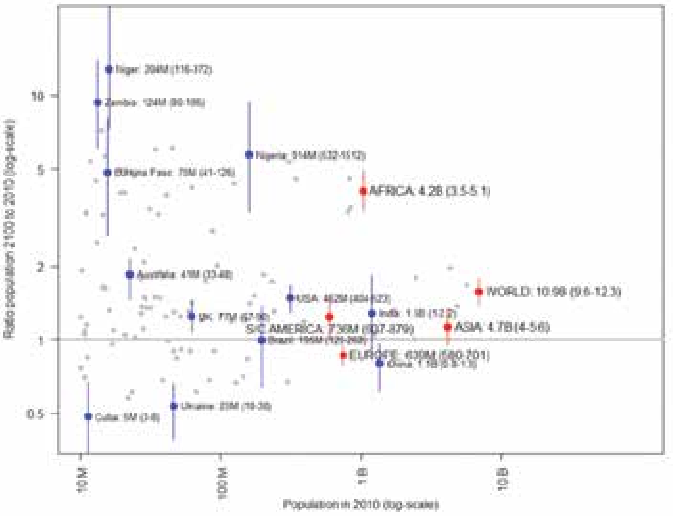


projections allows users of these projections to work not only with the "best guess" of future demographic outcomes but also to incorporate the associated uncertainty ranges in their analyses.

Figure 5 shows projected population growth for major areas and selected countries with current populations greater than 10 million. Plotted are the ratios of the projected population in 2100 to the population in 2010 (gray dots). The world and selected areas are highlighted, and 80% prediction intervals for the relative change from 2010 to 2100 are added to illustrate the projection uncertainty.

The figure illustrates the diversity in projected changes as well as in uncertainty surrounding the projections. Projections for all countries of the world and further details on

Figure 5. Population Projections for 2100



"B" denotes Billion and "M" denotes Million. Numbers shown with Area Names are the Population Projections for 2100 with 80% Prediction Intervals.

APPENDIX

The Use of Bayesian Hierarchical Models in Probabilistic Projections

In Bayesian inference, prior beliefs about unknown quantities are summarized in prior probability distributions and updated based on new information (data) to obtain the posterior probability distribution, which summarizes all available information about the quantity of interest. For example, when constructing country-specific TFR projections, one of the unknown parameters is the maximum decline in a five-year period that is expected to occur during the transition in Phase II. In Bayesian inference, beliefs about this quantity are represented with a probability distribution, and data on observed declines are used to update this distribution.

In the context of projections, however, the difficulty with parameter estimation is usually the lack of information on the parameters of interest. For the TFR projections, country-specific information on the expected maximum decline in a five-year period is limited for any country which has only just started its fertility decline. Bayesian hierarchical models provide a practical solution to this problem if information can be "shared" across populations.

The main idea in hierarchical models used for country-specific projections is to exchange information on unknown quantities across countries, based on the assumption that these quantities are drawn from a common probability distribution. For estimating the maximum decline in the TFR for each country, the hierarchical model is based on a world-level probability distribution that summarizes the mean, variability, and distribution of all country-specific outcomes. Countries that have finished or are far along in their fertility transitions provide information about this world-level probability distribution. Parameter estimates for countries which have only just started their transitions are informed by the world-level probability distribution. The resulting estimates for a specific country can thus be considered as being informed by past declines in that country as well as observed declines in other countries.

When using a Bayesian hierarchical model for parameter estimation, posterior probability distributions are obtained for all parameters of interest, although usually not in closed form. Instead, sampling algorithms such as the Markov Chain Monte Carlo (MCMC) algorithm are used to obtain samples from the posterior distributions. For the TFR projections, an MCMC algorithm was used to obtain samples of the parameters of the decline curve for each country.

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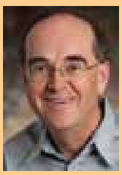

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the methodology for the UN probabilistic projections and can be found at ***http://esa.un.org/unpd/ppp/***.

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