Mathematical Aspects of 3D Photography

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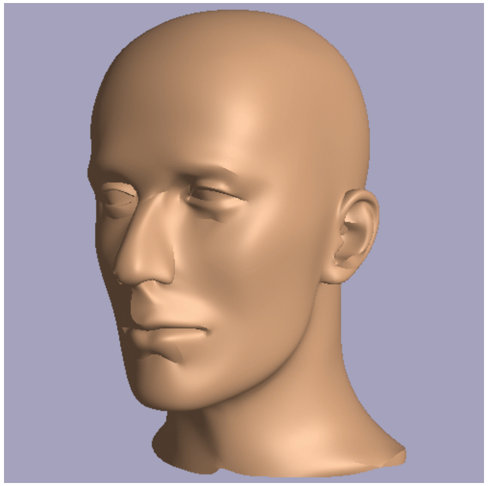
Funded by NSF and industry contributions

Outline of talk

- What is 3D Photography, and what is it good for ?
- Sensors
- Modeling 2D manifolds by subdivision surfaces
- Parametrization and multiresolution analysis of meshes
- Surface light fields
- Conclusions

1. What is 3D Photography and what is it good for ?



Emerging technology aimed at

- capturing
- viewing
- manipulating

digital representations of shape and visual appearance of 3D objects.

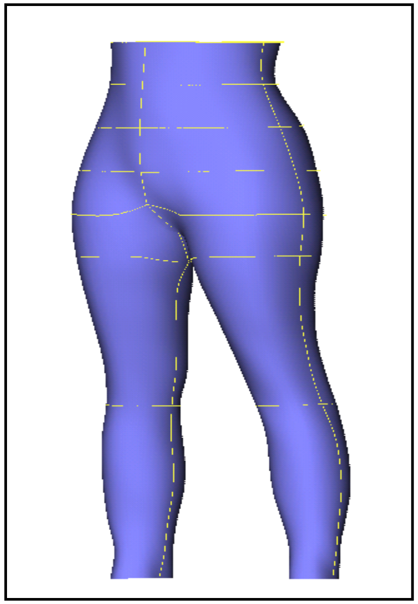
Will have large impact because 3D photographs can be

- stored and transmitted digitally,
- viewed on CRTs,
- used in computer simulations,
- manipulated and edited in software, and
- used as templates for making electronic or physical copies

Modeling humans

- Anthropometry
- Create data base of body shapes for garment sizing
- Mass customization of clothing
- Virtual dressing room
- Avatars

Scan of lower body
(Textile and Clothing Technology Corp.)



Fitted template
(Dimension curves drawn in yellow)

Full body scan
(Cyberware)

Modeling artifacts

- Archival
- Quantitative analysis
- Virtual museums

Image courtesy of Marc Levoy and the Digital Michelangelo project

Left: Photo of David's head
Right: Rendition of digital model

(1mm spatial resolution, 4 million polygons)

Modeling artifacts

Images courtesy of Marc Rioux and the Canadian National Research Council

Nicaraguan stone figurine

Painted Mallard duck

Modeling architecture

- Virtual walk-throughs and walk-arounds
- Real estate advertising
- Trying virtual furniture

Left image: Paul Debevec, Camillo Taylor, Jitendra Malik (Berkely)

Right image: Chris Haley (Berkeley)

Model of Berkeley Campanile

Model of interior with artificial lighting

Modeling environments

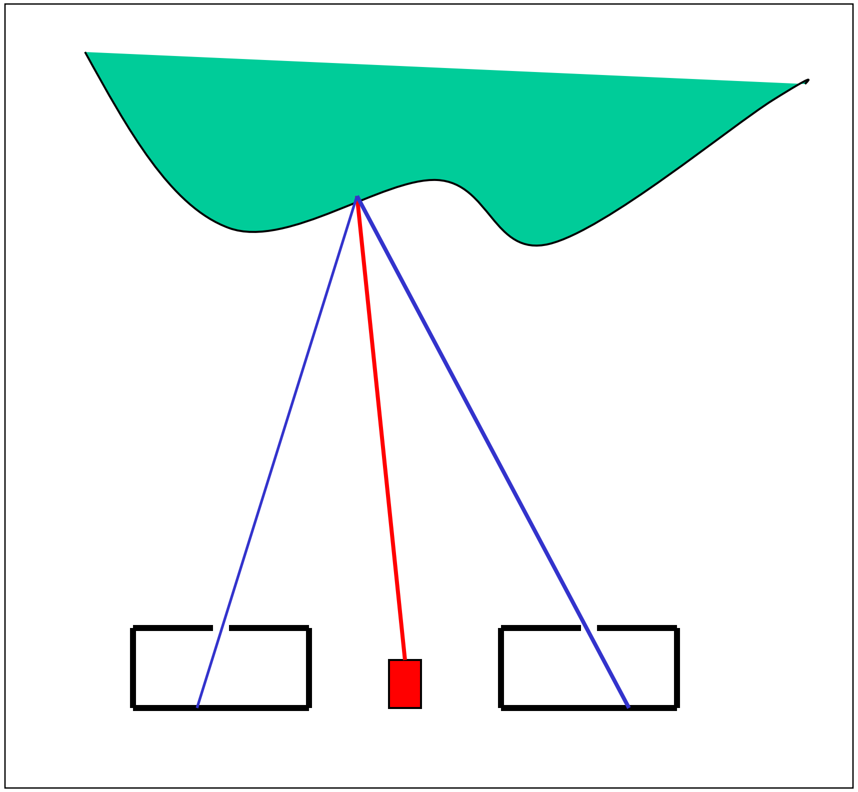
- Virtual walk-throughs and walk arounds
- Urban planning

Two renditions of model of MIT campus
(Seth Teller, MIT)

2. Sensors

Need to acquire data on shape and “color”

Simplest idea for shape: Active light scanner using triangulation



UW “handknit” scanner

Laser spot on object allows matching of image points in the cameras

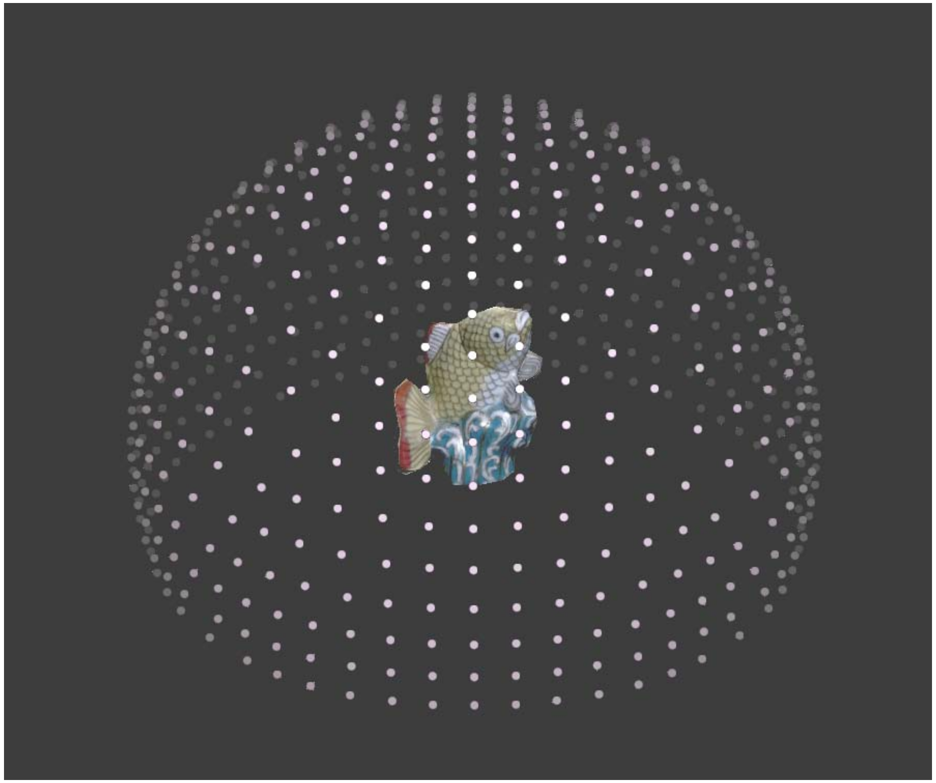
A more mature engineering effort:

The Cyberware Full Body Scanner

“Color” acquisition

“Color” can mean:

- RGB value for each surface point
- RBG value for each surface point and viewing direction
- BRDF (allows re-lighting)



One of ~ 700 images

Camera positions

Output of sensing process

- 1,000’s to 1,000,000’s of surface points assembled into triangular mesh
- RBG value for each vertex   or
- Collection of (direction, RGB value) pairs for each vertex

Mesh generated from fish scans

4. Modeling shape

A computer scientist's view

"Triangular mesh" is a basic abstraction in computer graphics and computational geometry.

Extensive set of tools for storing and manipulating meshes

Representing object surface by triangular mesh interpolating surface points comes natural to a computer scientist

A mathematician's view

Mathematical abstraction for surface of 3D object is "embedded 2D manifold" (subset of 3D space that locally looks like a piece of the plane)

Study of 2D manifolds has a long history going back to Gauss and Euler

Important result: There are infinitely many fundamentally different 2D manifolds that cannot be smoothly deformed into each other: impossible to deform balloon into coffee cup without tearing.

This fact accounts for some of the difficulties in 3D photography.

A statistician's view

We have a set of data - surface points produced by the sensor.

We want to "fit a parametric model" to these data, in our case a 2D manifold.

Parameters of model control shape of the manifold.

We define a goodness-of-fit measure quantifying how well model approximates data.

We then find the best parameter setting using numerical optimization.

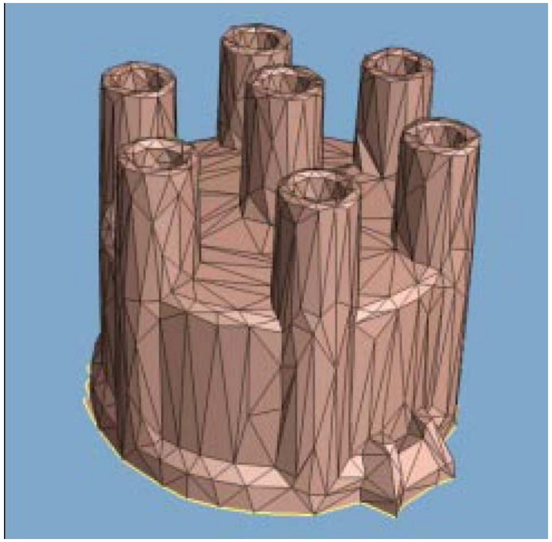
Basic questions:

- What's the form of the parametric model ?
- What's the goodness-of-fit measure ?
- (How will we optimize it ?)

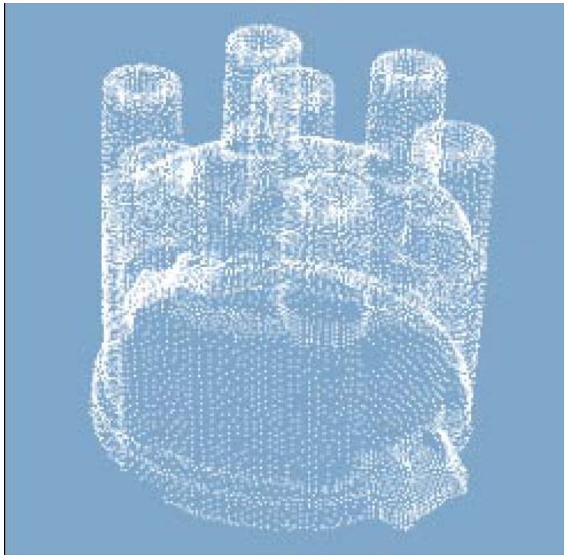
Fitting 2D manifolds

Why not stick with meshes ?

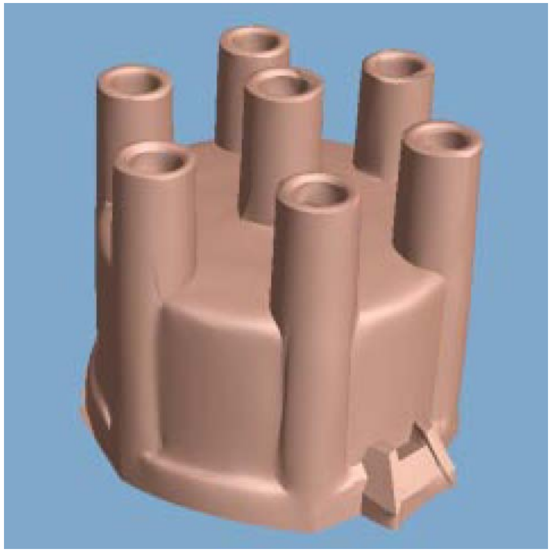
- Real world objects are often smooth or piecewise smooth
- Modeling a smooth object by a mesh requires lots of small faces
- Want more parsimonious representation



Fitted mesh



Sensor data



Fitted subdivision surface

Subdivision surfaces

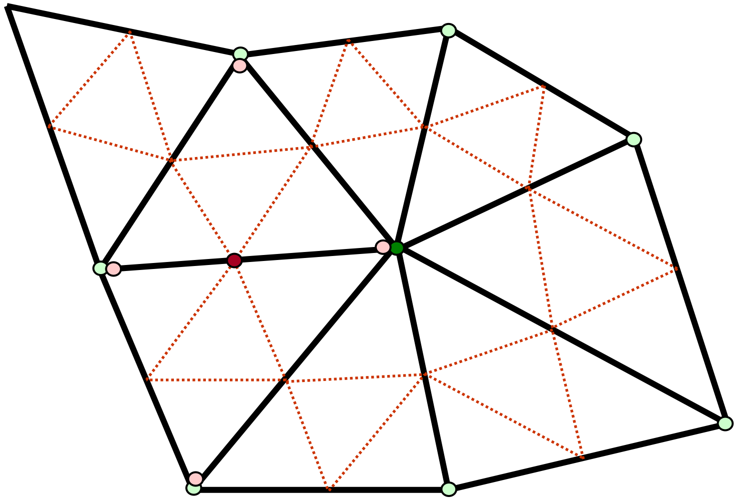
Defined by limiting process, starting with control mesh (bottom left)

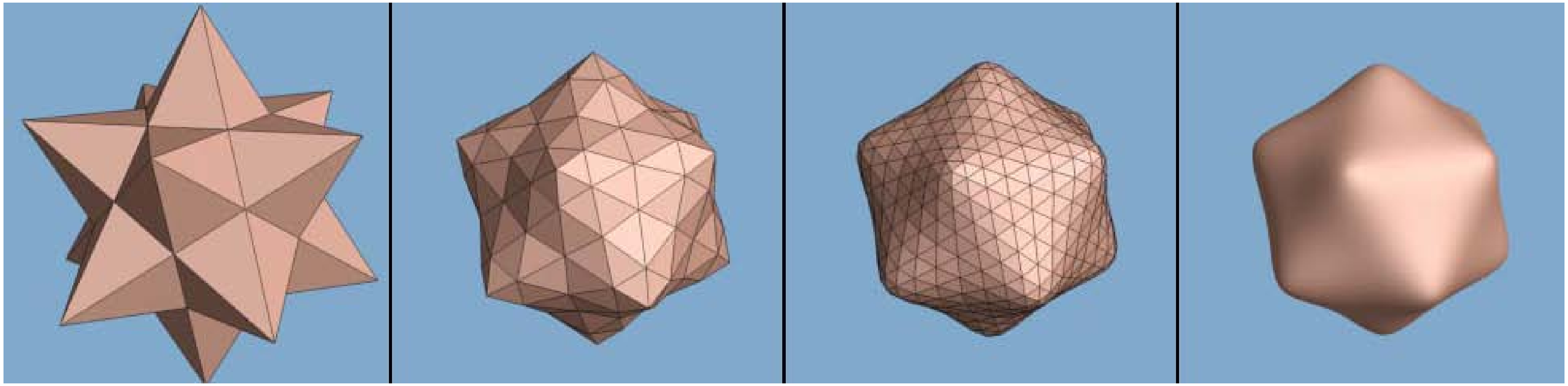
Split each face into four (right)

Compute positions of new **edge vertices** as weighted means of **corner vertices**

Compute new positions of **corner vertices** as weighted means of their **neighbors**

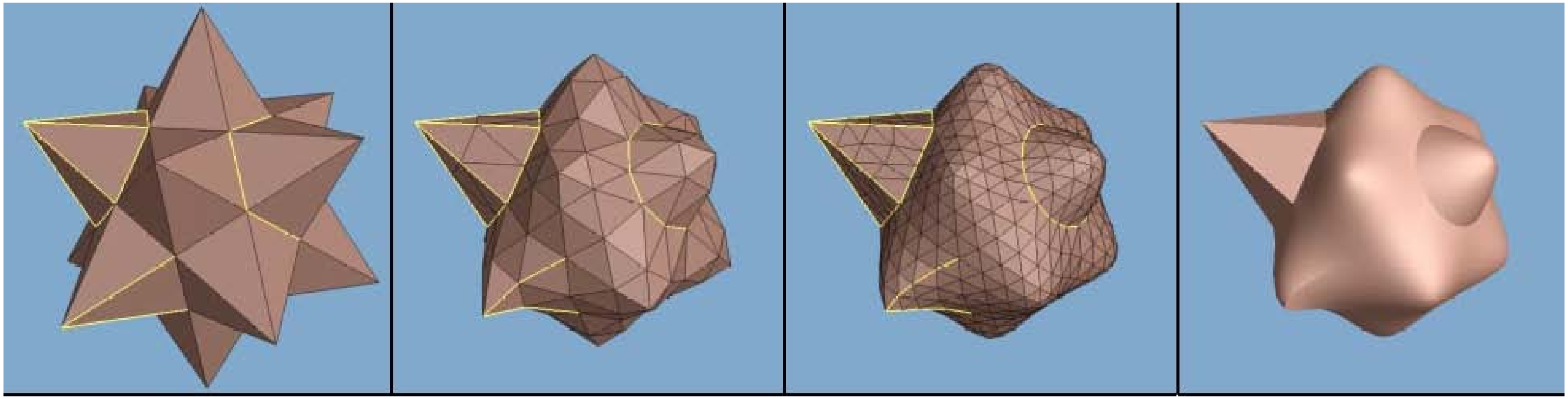
Repeat the process

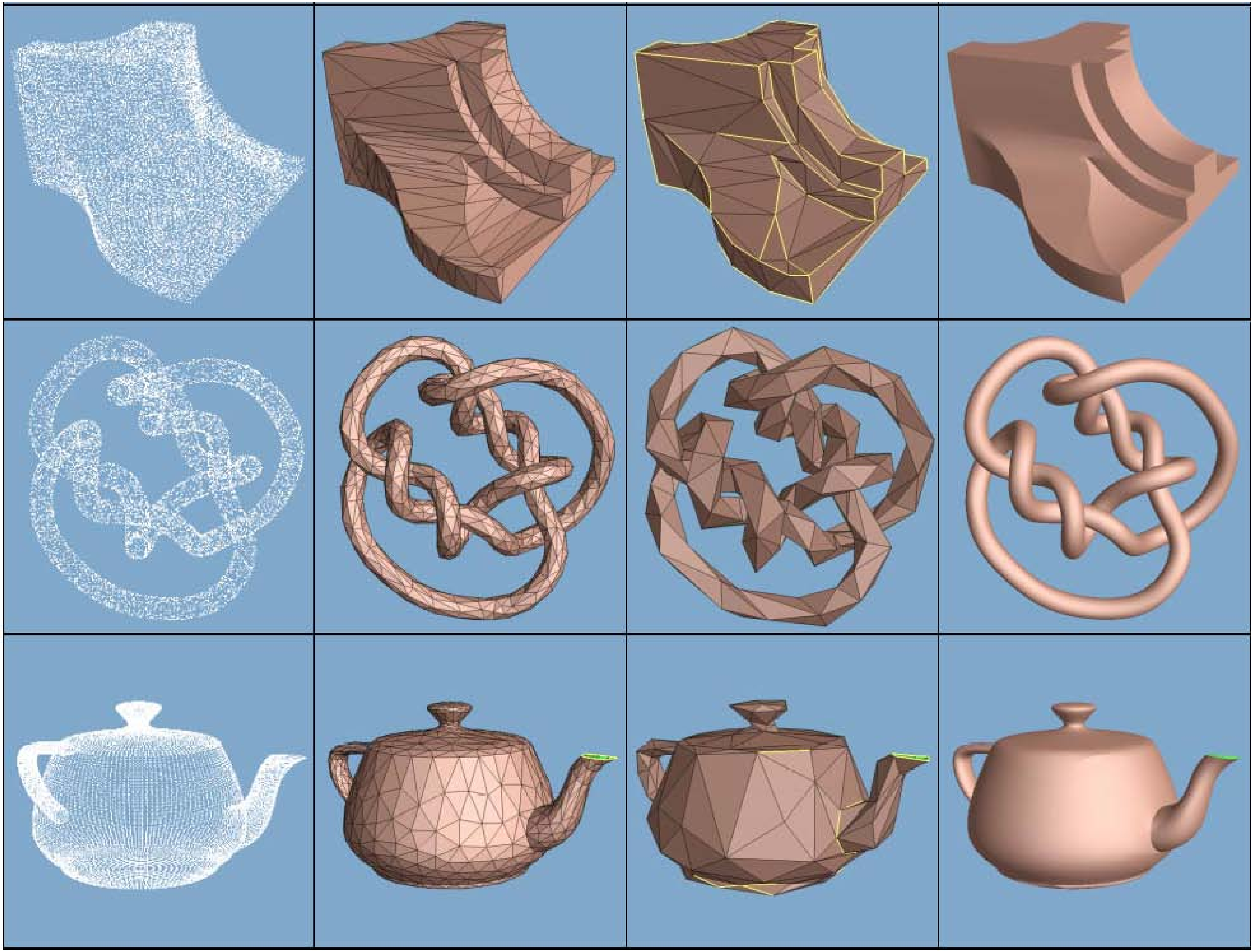




Remarks

- Limiting position of each vertex is weighted mean of control vertices.
- Important question: what choices of weights produce smooth limiting surface ?
- Averaging rules can be modified to allow for sharp edges, creases, and corners (below)
- Fitting subdivision surface to data requires solving nonlinear least squares problem.

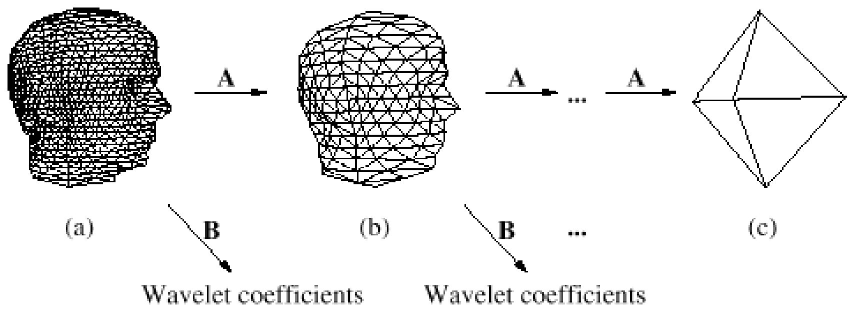




6. Parametrization and multiresolution analysis of meshes

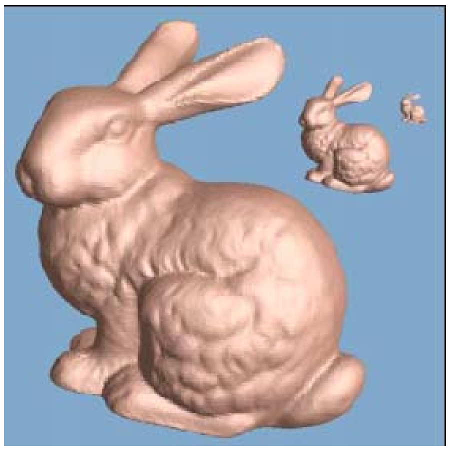
Idea:

Decompose mesh into simple “base mesh” (few faces) and sequence of “wavelet” correction terms of decreasing magnitude

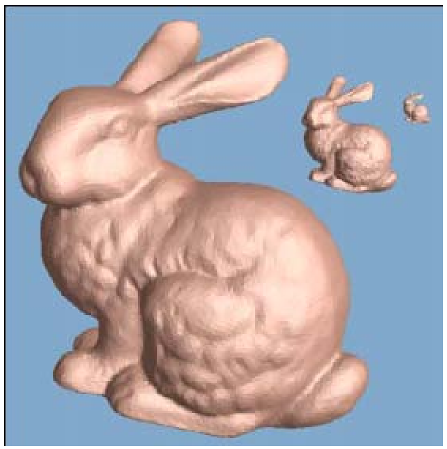


Motivation:

- Compression
- Progressive transmission
- Level-of-detail control
  - Rendering time ~ number of triangles
  - No need to render detail if screen area is small



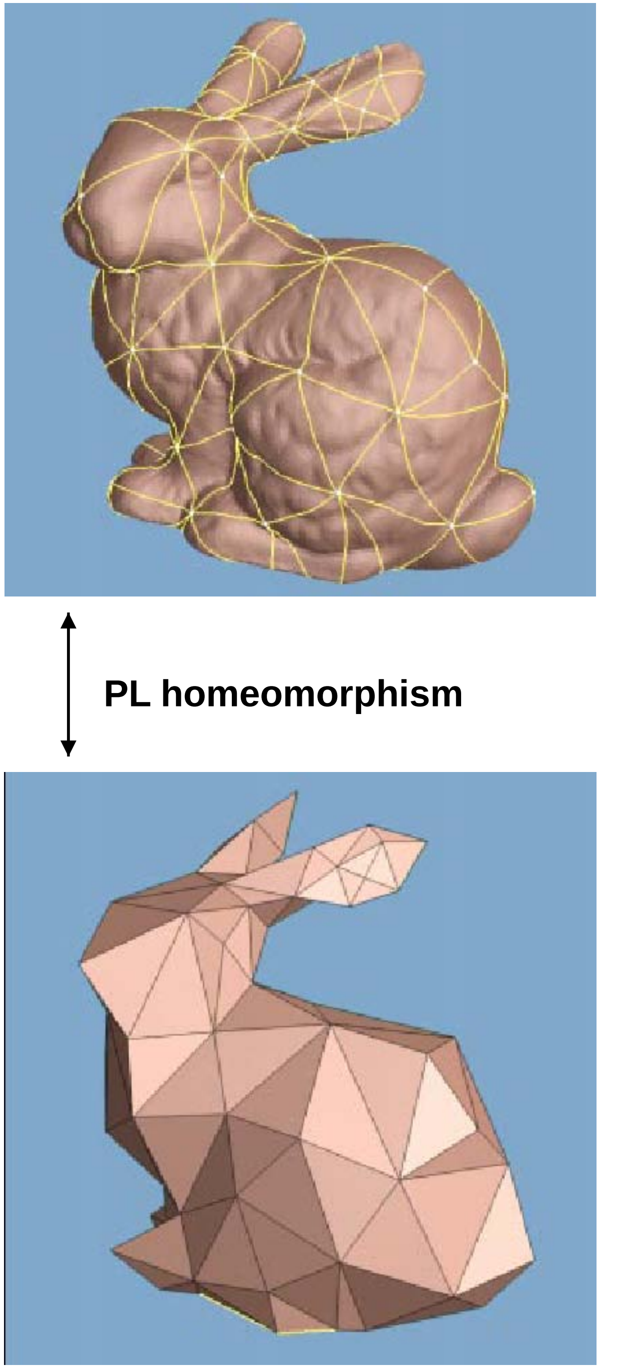
Full resolution
70K faces



LoD control
38K - 4.5K - 1.9K faces

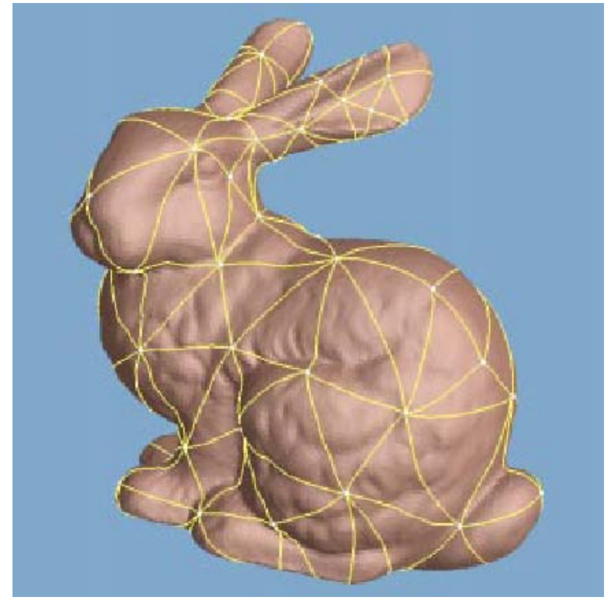
Procedure
(“computational differential geometry”)

- Partition mesh into triangular regions, each homeomorphic to a disk
- Create a triangular “base mesh”, associating a triangle with each of the regions
- Construct a piecewise linear homeomorphism from each region to the corresponding base mesh face
- Now we have representation of original as vector-valued function over the base mesh
- Multi-resolution analysis of functions is (comparatively) well understood.

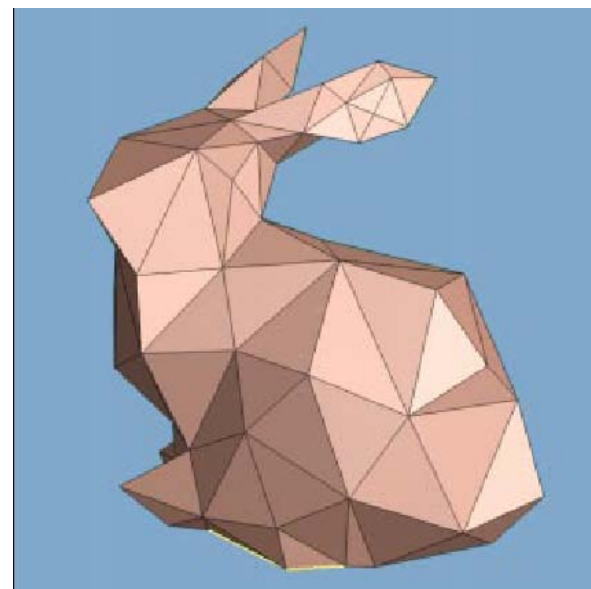


Texture mapping

- Homeomorphism allows us to transfer color from original mesh to base mesh
- This in turn allows us to efficiently color low resolution approximations (using texture mapping hardware)
- Texture can cover up imperfections in geometry



PL homeomorphism



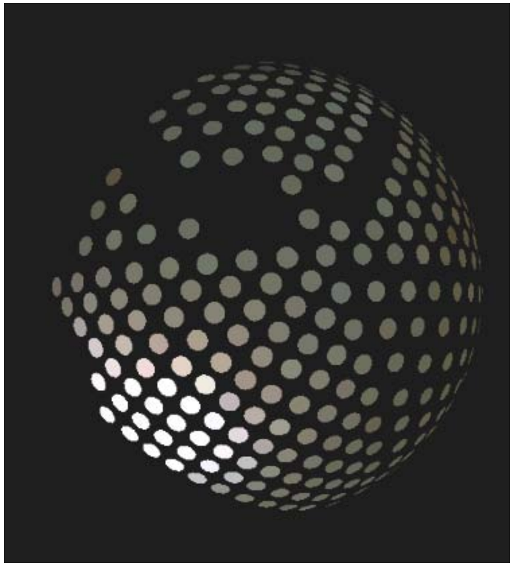
Mesh doesn’t much look like face, but…

What would it look like without texture ?

7. Modeling of surface light fields

Motivation

- Real objects don’t look the same from all directions (specularity, anisotropy)
- Ignoring these effects makes everything look like plastic
- Appearance under fixed lighting is captured by “surface light field” (SLF)
- SLF assigns RGB value to each surface point and each viewing direction - SLF is function assigning vector valued function on the sphere to each surface point.



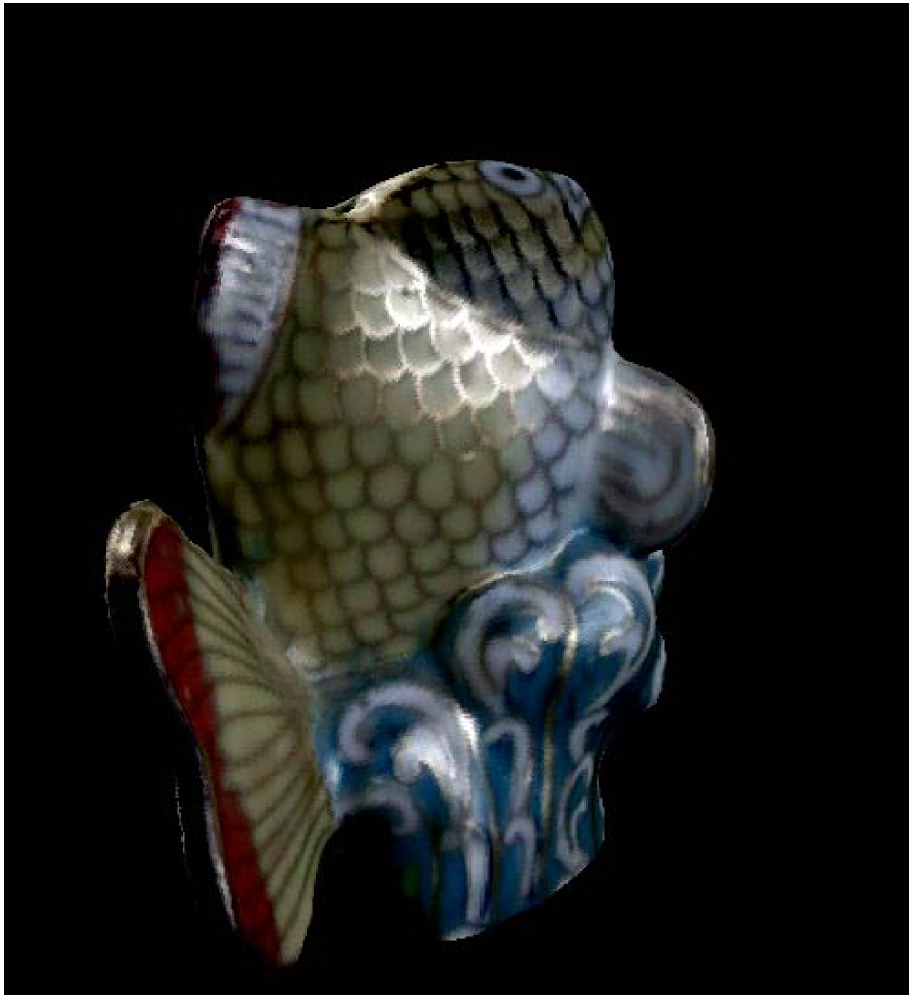
Data lumispere: observed direction - color pairs for single surface point

Payoff

Modeling and rendering SLF adds a lot of realism

Issues

- Compression: uncompressed SLF for fish is about 170 MB
- Real time rendering non-trivial
- Interesting mathematical / statistical problems: smoothing and approximation on general manifolds



8. Conclusions

3D Photography is an active, exciting research area

There is opportunity, and need, for contributions from Computer Science, Mathematics, and Statistics:

- Computer Scientists, Mathematicians, and Statisticians have a different ways of thinking about problems.
- Each discipline has evolved its own set of abstractions and created its own sets of tools.
- Casting 3D photography into the language of Mathematics and Statistics allows one to bring to bear the tools of these fields
- Thinking about 3D photography in mathematical or statistical terms suggests interesting research problems in those fields
- Broadening one's view through collaborative research is intellectually stimulating as well as enjoyable

Thank you for your patience

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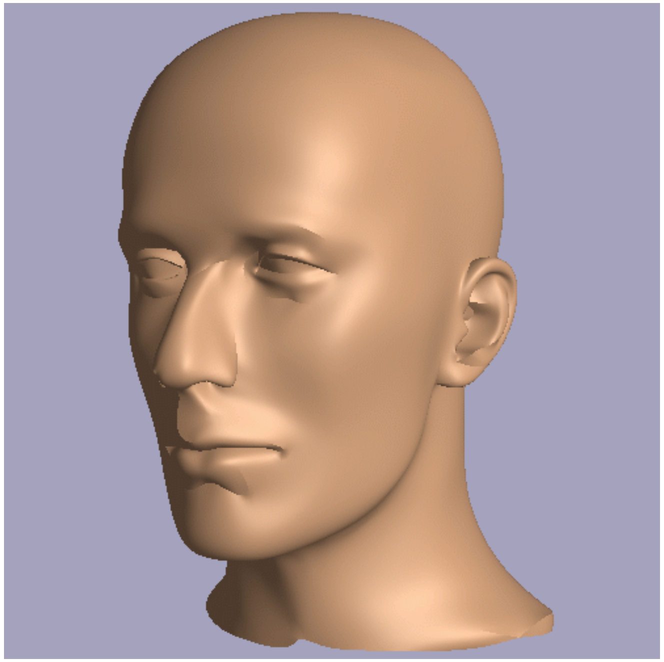

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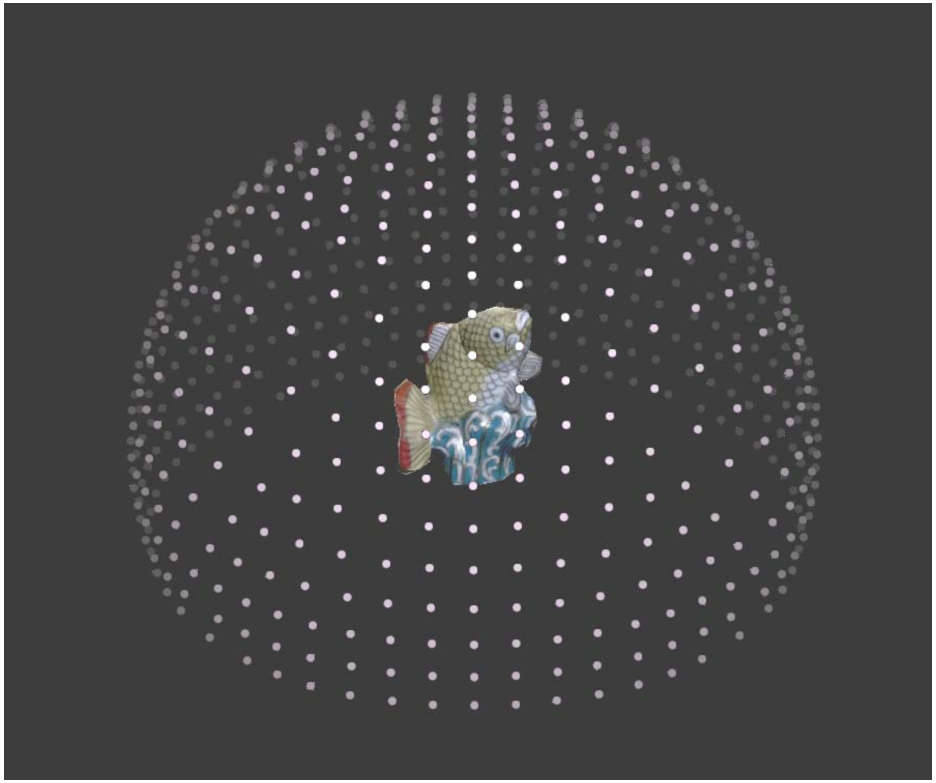

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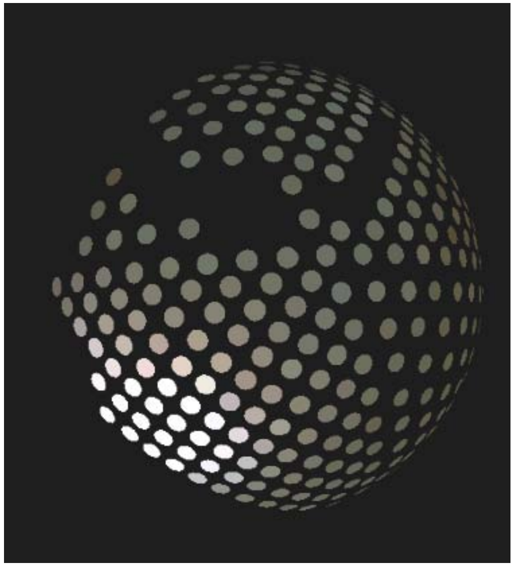

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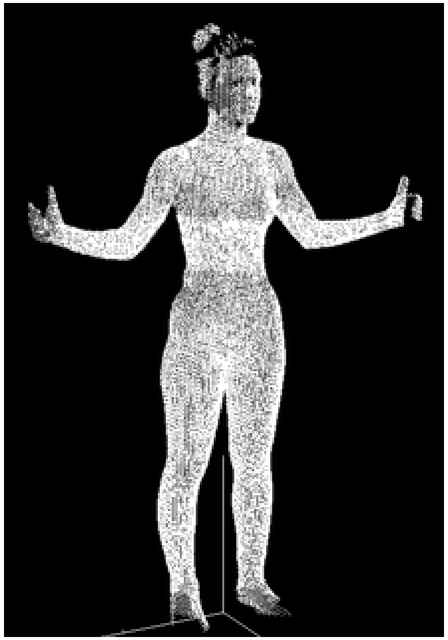
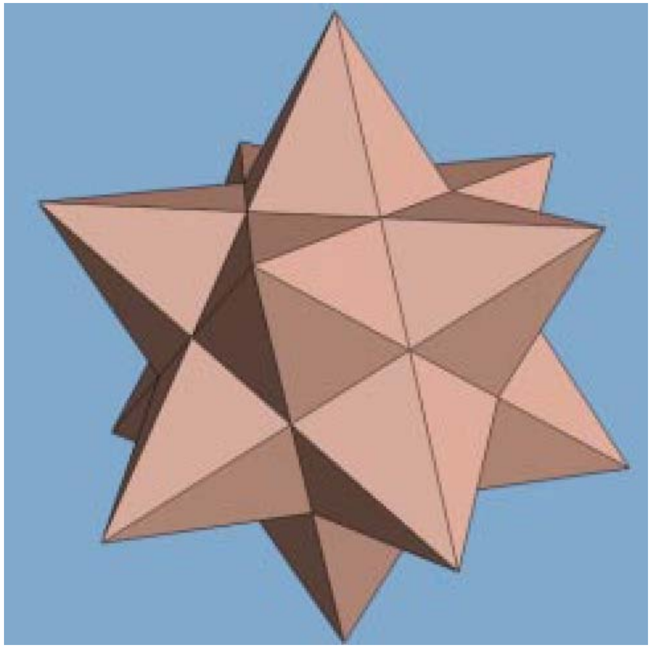
How would a mathematician think about

- The surface of a 3D object is a 2D manifold
- “Color” is a function assigning a 3D vector (RGB) to each point on a 2D manifold
- “Luminance”





3. Casting 3D photography into the language of Mathematics and Statistics

Why bother ?

- Computer Scientists, Mathematicians, and Statisticians have a different ways of thinking about problems.
- Each discipline has evolved its own set of abstractions and created its own sets of tools.
- Casting 3D photography into the language of Mathematics and Statistics allows us to bring to bear the tools of these fields.
- Thinking about 3D photography in mathematical or statistical terms might suggest interesting research problems in those fields - in fact is has.
- For the individuals involved, broadening the view has proven intellectually stimulating as well as enjoyable.

Will try to illustrate these points using a few examples.